☆ THE ART OF COMPUTER PROGRAMMING ☆
EARLIEST ERRATA TO VOLUME 3 (2nd edition)

This document is a transcript of the notes that I had been making in my personal copy of *The Art of Computer Programming*, Volume 3 (second edition) from the time it was first printed in 1998 until the 27th printing came out in 2011.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ——— 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \rightsquigarrow n+1$

Page 666 line 2 ——— 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ——— 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 3. I didn't do that in the second edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 3 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, future editions of Volumes 1–3 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about* The Art of Computer Programming *is posted on*

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, February 1998

Writing a series like The Art of Computer Programming
is similar to painting the Forth Rail Bridge.
No sooner is it finished than
the job must be started again.
— MALCOLM CLARK (1992)

The time when The Guardian *ceases to make mistakes altogether*
is not, at the moment, foreseeable.
— IAN MAYES (1998)

☆☆☆☆☆☆☆ SORTING AND SEARCHING ☆☆☆☆☆☆☆

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Last updated 16 August 2022

All of these corrections have been made in subsequent printings.

Page iv line 3 of the Library of Congress Data ——— 07 Jan 2011

`xiv,780p.` ⇝ `xiv,782p.`

Page vii lines 23 and 24 ——— 28 Oct 2008

cheerfully pay $2.56 ⇝ cheerfully award $2.56

Page ix line 11 ——— 09 Oct 2010

This is sometimes unfortunate ⇝ A motley mixture is, however, often unfortunate

Page x line −6 ——— 09 Oct 2010

This is not ⇝ This distinction is not

Page xi line 9 ——— 09 Oct 2010

Later editions ⇝ Later printings

Page xiii replacement for the bottom line ——— 07 Jan 2011

▸**Page 1** line 7 ——— 05 May 1998

The Prince (1951) ⇝ *The Prince* (1513)

Page 5 line 3 of exercise 3 ——— 16 Nov 1998

conditions (1) and (2) ⇝ conditions (1) and (2)

▸**Page 6** line 1 of exercise 9 ——— 24 Aug 2002

After n ⇝ After N

▸**Page 7** line 2 ——— 03 Jan 2000

at most ten ⇝ less than a dozen

Page 7 line −24 ——— 14 Sep 1998

Achtzehnhundert zwölf ⇝ Achtzehnhundertzwölf

Page 7 line −7 ——— 11 Nov 1998

süssen Mädeln. ⇝ langen Tag.

Page 8 line 31 ——— 16 Feb 2003

upper case letter ⇝ uppercase letter

Page 12 near the top ——— 19 Dec 2001

line 9: Marshall Hall's ⇝ the simple
line 11: [See *Proc.* . . . 203.] We ⇝ We

Page 14 line 7 — 09 Oct 2001

K. F. Hindenburg $\rightsquigarrow$ C. F. Hindenburg

▸**Page 15** line 2 after (7) — 28 Oct 2000

Rodriguez $\rightsquigarrow$ Rodrigues

▸**Page 17** line 3 — 12 May 1998

$\bigl((a_1\,a_2\ldots a_n),\ (p_1,p_2,\ldots,p_n)\bigr)$ $\rightsquigarrow$ $\bigl((a_1,a_2,\ldots,a_n),\ (p_1,p_2,\ldots,p_n)\bigr)$

▸**Page 17** line 6 — 12 May 1998

$\mathrm{ind}(a_1,a_2,\ldots,a_n)$ $\rightsquigarrow$ $\mathrm{ind}(a_1\,a_2\ldots a_n)$

Page 18 line 1 — 23 May 1998

5 4 6 1 3 8 7 2; $\rightsquigarrow$ 5 4 6 1 3 8 7 2 (man 1 is 5th out, etc.);

▸**Page 20** line 2 of exercise 19 — 09 Aug 1999

$((n-1)m \bmod m)$ $\rightsquigarrow$ $((n-1)m \bmod n)$

Page 22 line 1 of exercise 28 — 23 Nov 2002

(R. W. Floyd, 1983.) $\rightsquigarrow$

Page 23 lines −20 and −15 — 15 Aug 2003

Anuyogadvāra-sutra $\rightsquigarrow$ *Anuyogadvārasūtra*

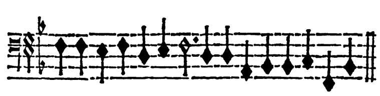
Page 23 near the bottom — 20 Jul 2000

line −12: *Líĺávatí* of Bháscara Áchárya $\rightsquigarrow$ *Līlāvatī* of Bhāskara
lines −11 and −8: Bháscara $\rightsquigarrow$ Bhāskara

Page 23 from line −4 and continuing into page 24 — 15 Oct 2004

The correct rule . . . A few years later, $\rightsquigarrow$

The correct rule for counting permutations when elements are repeated was apparently unknown in Europe until Marin Mersenne stated it without proof as Proposition 10 in his elaborate treatise on melodic principles [*Harmonie Universelle* **2**, also entitled *Traitez de la Voix et des Chants* (1636), 129–130]. Mersenne was interested in the number of tunes that could be made from a given collection of notes; he observed, for example, that a theme by Boesset,



can be rearranged in exactly $15!/(4!\,3!\,3!\,2!) = 756{,}756{,}000$ ways.

The general rule (3) also appeared in Jean Prestet's *Élémens des Mathématiques* (Paris: 1675), 351–352, one of the very first expositions of combinatorial mathematics to be written in the Western world. Prestet stated the rule correctly for a general multiset, but illustrated it only in the simple case $\{a,a,b,b,c,c\}$. A few years later,

Page 27 line −10 — 22 Aug 2002

Paris **258** (1964) $\rightsquigarrow$ **258** (Paris, 1964)

Page 31 line 2 of exercise 10 ——— 15 Jun 2003

insure $\rightsquigarrow$ ensure

▸**Page 31** line −5 ——— 20 Jul 2001

notion of $\rightsquigarrow$ notion of "topological sorting"

▸**Page 32** line 3 of exercise 15 ——— 13 Aug 2002

$x_1 < x_2 < \cdots < x_n$ and $n_1 + n_2 + \cdots + n_m = m$ $\rightsquigarrow$ $x_1 < x_2 < \cdots < x_m$ and $n_1 + n_2 + \cdots + n_m = n$

Page 33 line 8 of exercise 19 ——— 01 Jan 2006

$\pi = x_{j_1}\, x_{j_2} \ldots x_{j_n}$, where $1 \le j_k \le m$ $\rightsquigarrow$ $\pi = x_{i_1}\, x_{i_2} \ldots x_{i_n}$, where $1 \le i_k \le m$

▸**Page 35** line 2 ——— 01 Jan 2006

the largest x_j $\rightsquigarrow$ the largest j

Page 35 line 7 ——— 01 Jan 2006

$\circledR$ is simply juxtaposition; $\rightsquigarrow$ $\circledR$ sorts $\{w_1, \ldots, w_k, x_1, \ldots, x_l\}$, but it simply juxtaposes $y_1 \ldots y_k$ with $z_1 \ldots z_l$;

Page 35 replacement for the displayed equation on line 10 ——— 01 Jan 2006

$$\pi = (x_{11} \ldots x_{1n_1}\, y_1) \circledR \bigl((x_{21} \ldots x_{2n_2}\, y_2) \circledR (\cdots \circledR (x_{t1} \ldots x_{tn_t}\, y_t) \cdots)\bigr)$$

Page 35 line 11 ——— 01 Jan 2006

by letting (11) stand for $\rightsquigarrow$ by letting the two-line array (11) correspond to

▸**Page 39** line 5 ——— 13 Jun 2006

Magazine **33** $\rightsquigarrow$ *Magazine* **32**

▸**Page 40** lines 10 and 11 from the bottom ——— 18 May 2000

$$+\sum_{j=0}^{k} \rightsquigarrow +\sum_{j=0}^{k-1} \qquad \text{(twice)}$$

▸**Page 41** replacement for the bottom line ——— 19 May 2000

$$R_m(z) = L(z) + \frac{2z}{z-z_0} + \frac{z}{z-z_1} + \frac{z}{z-\bar z_1} + \frac{z}{z-z_2} + \frac{z}{z-\bar z_2} + \cdots + \frac{z}{z-z_m} + \frac{z}{z-\bar z_m}$$

▸**Page 42** replacement for line 5 ——— 19 May 2000

$$\frac{2z}{1-z} + \frac{z/z_1}{1-z/z_1} + \frac{z/\bar z_1}{1-z/\bar z_1} + \cdots + \frac{z/z_m}{1-z/z_m} + \frac{z/\bar z_m}{1-z/\bar z_m} + R_m(z),$$

▸**Page 42** line 3 after (31) ——— 19 May 2000

$R_m(z) \to -z$ $\rightsquigarrow$ $R_m(z) \to cz$ for some constant c

Page 42 line 4 after (31) ——— 26 May 2000

when $n > 1$. $\rightsquigarrow$ when $n > 1$. (See also exercise 28.)

▸**Page 43** in Figure 3 ——— 01 Jan 2006

[the x axis, namely the horizontal line $y = 0$, should be thicker]

Page 45 replacement for line 3 — 18 May 2000

$$\sum_k \left\langle {n \atop k} \right\rangle \binom{k}{n-q} = \left\{ {n \atop q} \right\} q!, \qquad \text{integer } q \geq 0.$$

Page 46 line 3 — 29 Jan 2005

alternatively $\rightsquigarrow$ alternately

Page 46 exercise 16 — 14 Aug 2002

[change the notation from $\left| {n \atop k} \right|$ to $\big\wr {n \atop k} \big\wr$, because I'm now using the former notation for partition numbers in Volume 4]

Page 46 exercise 19 — 26 Nov 2003

J. Riordan $\rightsquigarrow$ I. Kaplansky and J. Riordan, 1946

▸**Page 47** line 1 of exercise 27 — 01 Jan 2006

is a forest $\rightsquigarrow$ is an oriented forest

Page 47 new exercise — 26 May 2000

28. [*HM35*] Find the asymptotic value of the numbers z_m in Fig. 3 as $m \to \infty$, and prove that

$$\sum_{m=1}^{\infty} (z_m^{-1} + \bar{z}_m^{-1}) \;=\; e - \frac{5}{2}.$$

Page 47 another new exercise — 29 Jan 2005

▸**29.** [*M30*] The permutation $a_1 \ldots a_n$ has a "peak" at a_j if $1 < j < n$ and $a_{j-1} < a_j > a_{j+1}$. Let s_{nk} be the number of permutations with exactly k peaks, and let t_{nk} be the number with k peaks *and* k descents. Prove that (a) $s_{nk} = \frac{1}{2}\big\wr {n \atop 2k} \big\wr + \big\wr {n \atop 2k+1} \big\wr + \frac{1}{2}\big\wr {n \atop 2k+2} \big\wr$ (see exercise 16); (b) $s_{nk} = 2^{n-1-2k} t_{nk}$; (c) $\sum_k \left\langle {n \atop k} \right\rangle x^k = \sum_k t_{nk} x^k (1+x)^{n-1-2k}$.

▸**Page 52** line 16 — 12 May 1998

poof $\rightsquigarrow$ proof

Page 54 line 4 — 01 Jan 2006

array (16); interchanging $\rightsquigarrow$ array (16); its columns are essentially independent and can be rearranged. Interchanging

Page 54 line -10 — 06 Jan 2011

Corollary. $\rightsquigarrow$ **Corollary B.**

▸**Page 59** line -6 — 01 Jan 2006

in much $\rightsquigarrow$ in a much

▸**Page 60** line 4 after the caption — 25 Nov 1998

$(n_2 + m - 1) \rightsquigarrow (n_2 + m - 2)$

Page 61 line -6 — 01 Jan 2006

$a_j > a_k > a_i$ for $i < j < k \rightsquigarrow a_i > a_k > a_j$ for $i < j < k$

▸**Page 63** line 1 of (46) — 17 May 1999

$\frac{1}{2} \ln n \rightsquigarrow \frac{1}{2} n \ln n$

▸**Page 63** line 2 of (49) ——— 09 Aug 1999

$\dfrac{x^6}{n^2} \leadsto \dfrac{8}{9}\dfrac{x^6}{n^2}$

Page 64 line 2 above (52) ——— 01 Jan 2006

$O\bigl(\exp(-2n^\epsilon)\bigr)$ in each term $\leadsto$ $O\bigl(\exp(-2n^{2\epsilon})\bigr)$ in each term

Page 64 line −6 ——— 22 Aug 2002

Paris (1782) $\leadsto$ (Paris, 1782)

Page 67 replacement for line −4 ——— 01 Jan 2006

$$17!/(12\cdot 11\cdot 8\cdot 7\cdot 5\cdot 4\cdot 1\cdot 9\cdot 6\cdot 5\cdot 3\cdot 2\cdot 5\cdot 4\cdot 2\cdot 1\cdot 1)$$

[and similarly, insert parentheses into the denominator in exercise 20]

Page 68 exercise 23 ——— 20 Aug 2007

$A_n \leadsto E_n$ (twice)

Page 69 exercise 32 ——— 09 Feb 2005

t_n is $\leadsto$ the involution number t_n is

Page 70 line 6 ——— 03 Jan 2003

outdegree $\leadsto$ out-degree

▸**Page 70** line 12 ——— 01 Jan 2006

columns sets $\leadsto$ column sets

Page 74 hyphenation on lines −8 and −7 ——— 09 Oct 2007

rear-ranged $\leadsto$ re-arranged

▸**Page 76** line −15 ——— 21 Jul 1998

of other $\leadsto$ of the other

▸**Page 77** top line ——— 07 Jun 1998

Table 2 $\leadsto$ **Table 1**

Page 77 line −4 ——— 03 Jun 1998

$K_1, \ldots, K_N \leadsto K_1 \ldots K_N$

▸**Page 82** replacement for line 11 ——— 01 May 1998

$B = \bigl(\min 0,\ \text{ave}\ (N^2-N)/4,\ \max\ (N^2-N)/2,\ \text{dev}\ \sqrt{N(N-1)(N+2.5)}/6\bigr);$

▸**Page 84** line 4 after the table ——— 23 May 1998

that w sort $\leadsto$ that we sort

Page 88 line −10 ——— 06 Jan 2011

Corollary. $\leadsto$ **Corollary H.**

▸**Page 90** line 11 ——— 09 Aug 1998

$h_t, \ldots, h_1 \leadsto h_{t-1}, \ldots, h_0$

▸**Page 92** in (9) — 22 May 2000

$\binom{r-1}{2} \le s \le \binom{r}{2} \rightsquigarrow \binom{r-1}{2} \le s < \binom{r}{2}$

▸**Page 92** line 1 of Theorem I — 09 Dec 1998

$O(Ne^{c\sqrt{\ln n}}) \rightsquigarrow O(Ne^{c\sqrt{\ln N}})$

Page 94 new entry before line −3 of Table 6 — 11 Apr 2001

190 84 37 16 7 3 1 359 7201 7

Page 94 near the bottom — 11 Apr 2001

line −5: $10NT \rightsquigarrow 10(NT - S)$

bottom three lines: Therefore ... first pass. $\rightsquigarrow$ (The first pass is very quick, however, if h_{t-1} is near N, because $NT - S = (N - h_{t-1}) + \cdots + (N - h_0)$.)

Page 95 lines 6–9 — 11 Apr 2001

He also discovered ... order $N^{3/4}$. $\rightsquigarrow$ On the other hand, subsequent tests by Marcin Ciura show that Sedgewick's sequence (11) apparently makes $B_{\text{ave}} = O\big(N(\log N)^2\big)$ or better. The standard deviation for sequence (11) is amazingly small for $N \le 10^6$, but it mysteriously begins to "explode" when N passes 10^7.

Page 95 near the bottom — 11 Apr 2001

in (12): $3h_t \ge N \rightsquigarrow h_{t+1} > N$

lines −6 to −4: recommended, again ... 10% faster. $\rightsquigarrow$ recommended. Still better results, possibly even of order $N \log N$, have been reported by N. Tokuda using the quantity $\lfloor 2.25h_s \rfloor$ in place of $3h_s$ in (12); see *Information Processing 92* **1** (1992), 449–457.

Page 98 line −1 — 26 Dec 2000

discussed below in $\rightsquigarrow$ discussed in

▸**Page 100** line −14 — 30 Apr 2008

`LD3 INPUT,1(1:1)` $\rightsquigarrow$ `LD4 INPUT,1(1:1)`

▸**Page 102** line 2 of exercise 7 — 12 May 1999

$|a_2 - 1| \rightsquigarrow |a_2 - 2|$

▸**Page 103** line 2 of exercise 17 — 24 Jan 2005

$\{1, 2, \ldots, n\} \rightsquigarrow \{1, 2, \ldots, N\}$

▸**Page 105** line 4 — 22 May 2000

the running time $\rightsquigarrow$ the average running time

Page 105 line 2 of exercise 40 — 01 Nov 2002

(15) $\rightsquigarrow$ (15)

▸**Page 105** line 4 of exercise 42 — 22 May 2000

$N/g \rightsquigarrow N^{3/2}/g$

Page 110 caption to Figure 16 is not an error — 22 Aug 2002

cocktail-shaker short [shic] $\leadsto$ cocktail-shaker short [shic]

[Let's drink a toast to all alcoholic shorting methods, and forgive authors who occasionally make weird attempts at humor.]

Page 111 step M3 of Algorithm M — 19 May 2005

'$\wedge$' $\leadsto$ '&' (six changes)

▸**Page 116** line 2 of step Q7 — 01 May 1998

$r + 1$. If $\leadsto$ $r + 1$.) If

Page 118 line 53 of the program — 27 Jul 1998

$j - N \leadsto j - N.$

Page 119 lines 21 and 22 — 24 Apr 2007

of the keys $K_1, \ldots, K_s$ $\leadsto$ of the first s keys $\{K_1, \ldots, K_s\}$

Page 121 replacement for line 2 of (25) — 15 Aug 1998

$$B_N = \tfrac{1}{6}\,(N+1)\bigl(2H_{N+1} - 2H_{M+2} + 1 - 6/(M+2)\bigr) + \tfrac{1}{2},$$

▸**Page 122** line 7 — 22 May 2000

Exercise 58 $\leadsto$ Exercise 42

▸**Page 125** line 1 of step R2 — 15 Aug 1998

$R_l \le \cdots \le R_r \leadsto R_l \ldots R_r$

▸**Page 125** line −3 — 15 Aug 1999

[rI4 $= l - j$] $\leadsto$ [rI4 $= j - l$]

▸**Page 126** lines *30* and *31* of the program — 15 Aug 1998

[rI4unknown] $\leadsto$ [rI4 unknown]
$b \leftarrow b - 1 \leadsto b \leftarrow b + 1$

▸**Page 127** line −15 — 14 Sep 2000

$\frac{1}{4}(\alpha+1)N \leadsto \frac{1}{2}N$

▸**Page 130** line 1 — 21 Sep 2001

$f_{-1} \leadsto f_{-1}(x)$

Page 136 new sentence added to exercise 28 — 25 Oct 1998

Ignore the comparisons made when computing the median value s.

▸**Page 137** line 2 of exercise 41 — 28 Oct 2000

$1 \le k < i \leadsto l \le k < i$

▸**Page 138** line 2 of exercise 55 — 05 Aug 1999

three keys (28). $\leadsto$ three keys (28), assuming that $M > 1$.

▸**Page 141** line −18 — 23 May 1998

$\{170, 175\} \leadsto \{170, 275\}$

▸**Page 148** line 4 — 14 Sep 2000

$\ln N$ ⇝ $\lg N$

▸**Page 151** line −7 — 01 May 1998

heaps). ⇝ heaps.)

▸**Page 152** line 9 — 09 Jan 2000

CACM **12** ⇝ *CACM* **21**

Page 152 line 14 — 29 Jul 2000

249 ⇝ 249; M. L. Fredman, *JACM* **46** (1999), 473–501

Page 152 lines 24–25 — 20 Mar 2000

SODA **8** (1997), 83–92] ⇝ *SICOMP* **28** (1999), 1326–1346]

▸**Page 155** line 2 — 14 Sep 2000

04805− ⇝ 04806−

Page 163 line −20 — 01 Jan 2002

regardless ⇝ or in the trivial case $N = 1$, regardless

▸**Page 166** line 4 after the program — 04 Dec 2005

about $9N \lg N$ ⇝ about $(8N \lg N)u$

▸**Page 166** line −12 — 29 Jul 1998

111.] ⇝ 111].

▸**Page 166** line −8 — 13 Aug 1998

arrangement ⇝ arrangements

▸**Page 166** line −3 — 31 Jul 1998

$K'_1 < \cdots K'_N$ ⇝ $K'_1 < \cdots < K'_N$

Page 166 line −1 — 11 May 1999

key appears ⇝ keys appear

Page 168 line 1 of exercise 18 — 16 Feb 2000

on N records ⇝ of N records

Page 174 line 07 of the program — 05 Oct 1999

$\rightarrow$ `TOP[`i`]` ⇝ $\rightarrow$ `TOP[`i`].`

▸**Page 177** line 1 — 20 Nov 1999

sort if ⇝ sort it

Page 177 line −1 — 10 Aug 1998

signed-magnitude ⇝ *signed magnitude*

▸**Page 183** line 23 — 09 Aug 1998

$\lceil \ln n! \rceil$ ⇝ $\lceil \lg n! \rceil$

Page 183 line 3 before (5) ——— 10 Apr 2009

begin as if we were sorting four elements by merging, first $\leadsto$ begin by first

Page 184 line 2 after (6) ——— 05 Feb 2009

two more $\leadsto$ one or two more

▸**Page 188** row b and column e of (24) ——— 16 Jun 1998

9 $\leadsto$ 7

▸**Page 189** in Eq. (26) ——— 28 Oct 2000

$$\frac{n'!}{2^{k'}T(G')}\frac{n''!}{2^{k''}T(G')} \leadsto \frac{n'!}{2^{k'}T(G')}\frac{n''!}{2^{k''}T(G'')}$$

Page 192 lines 3–5 ——— 02 Jan 2007

The intermediate ... takes no fewer comparisions $\leadsto$ Marcin Peczarski [see *Algorithmica* **40** (2004), 133–145; *Information Proc. Letters* **101** (2007), 126–128] extended Wells's method and proved that $S(13) = 34$, $S(14) = 38$, $S(15) = 42$, $S(22) = 71$; thus merge insertion is optimum in those cases as well. Intuitively, it seems likely that $S(16)$ will some day be shown to be less than $F(16)$, since $F(16)$ involves no fewer steps

Page 193 lines 2 and 3 after the figure ——— 14 Apr 2006

(This result ... 1956.) $\leadsto$ [This result was first obtained by A. Gleason in an internal IBM memorandum (1956).]

Page 194 line 1 ——— 22 Sep 2007

When $\leadsto$ (*Weak orderings.*) When

Page 197 exercise 35 ——— 20 Aug 2002

$S(13) \leadsto S(14)$

Page 203 in steps H1–H5 ——— 06 Jan 2011

[Summarize each step, as in the usual format for algorithms.]

▸**Page 204** line 2 ——— 05 Oct 1999

$M(m,n{-}2^t) \leadsto H(m,n{-}2^t)$

▸**Page 212** bottom line ——— 04 Jan 1999

acieves $\leadsto$ achieves

▸**Page 213** line −4 ——— 23 Dec 2002

exercise 26 $\leadsto$ exercise 27

▸**Page 224** line 4 before (3) ——— 21 Sep 2001

$\langle y_1,\ldots,y_m\rangle \leadsto \langle y_1,\ldots,y_n\rangle$

▸**Page 225** line −13 ——— 02 Jun 1998

needed in an $\leadsto$ needed in a

Page 225 line −8 ——— 21 Jan 2000

still unknown. $\rightsquigarrow$ known only in a very weak sense.

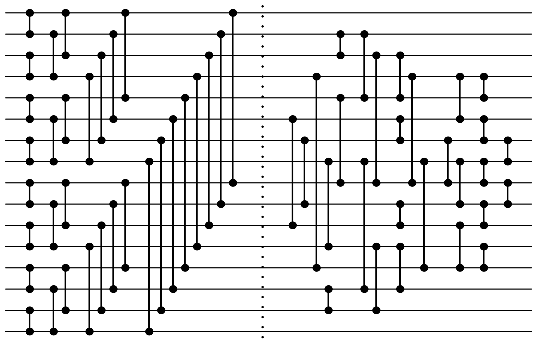
▸**Page 225** line −5 ——— 09 Oct 2009

D. J. O'Connor $\rightsquigarrow$ D. G. O'Connor

▸**Page 226** line −7 ——— 30 Jun 1999

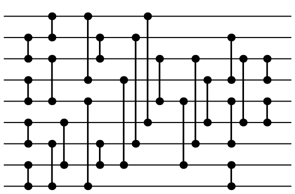
Hughes $\rightsquigarrow$ Hugues

Page 227 replacement for bottom part of Fig. 49 ——— 10 Aug 1998

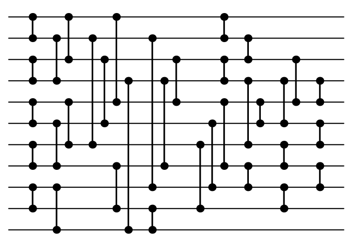


$n = 16$ 60 modules, delay 10

Page 229 new sorting networks for Fig. 51 ——— 11 Oct 2001



$n = 9$ 25 modules, delay 7



$n = 11$ 35 modules, delay 8

Page 229 line −10 ——— 11 Oct 2001

$n = 6$ and $n = 9$ $\rightsquigarrow$ $n = 6$, $n = 9$, and $n = 11$

Page 229 replacement for the bottom two lines ——— 11 Oct 2001

they always sort. Some of these networks were discovered in 1969–1971 by G. Shapiro ($n = 6$, 12) and D. Van Voorhis ($n = 10$, 16); the others were found in 2001 by Loren Schwiebert, using genetic methods ($n = 9$, 11).

▸**Page 230** line 22 ——— 21 Sep 2001

$\{z_{2n+1}, \ldots, z_{4n}\rangle$ to $\{z_1, \ldots, z_{2n}\rangle$ $\rightsquigarrow$ $\{z_{2n+1}, \ldots, z_{4n}\}$ to $\{z_1, \ldots, z_{2n}\}$

Page 231 line −5 ——— 23 Dec 2002

(m, n) merging $\rightsquigarrow$ (m, n)-merging

Page 232 line −22 — 24 Jan 2003

$(1,n)$ merging ⤳ $(1,n)$-merging

Page 235 notation change, four lines before exercise 1 — 08 Aug 2005

$x \le y$ ⤳ $x \subseteq y$ (twice)

Page 238 replacement for exercise 20 — 12 Sep 2003

20. [*28*] Prove that (a) $\hat{V}_3(5) = 7$; (b) $\hat{U}_4(n) \le 3n - 10$ for $n \ge 6$.

Page 238 notation change in line 2 of exercise 22 — 08 Aug 2005

$x \le y$ implies that $x\alpha \le y\alpha$ ⤳ $x \subseteq y$ implies that $x\alpha \subseteq y\alpha$

▸**Page 240** line −5 — 27 Aug 1998

$r \le 4n^2 + \sqrt{n}\lg n$ ⤳ $r \le 4n^2 + O(n^{3/2}\log n)$

Page 241 line 1 of exercise 43 — 23 Dec 2002

(m,n) merging ⤳ (m,n)-merging

▸**Page 241** two lines above Fig. 59 — 30 Aug 2004

replace comparisons by m-way merges. ⤳ replace each comparison by a merge network with $\hat{M}(m,m)$ modules.

Page 242 lines −3, −13, −17 — 17 Aug 1998

Fig. 61 ⤳ Fig. 60
Fig. 61 ⤳ **Fig. 60**

▸**Page 246** line 2 of exercise 66 — 05 Oct 1999

exercise 63 ⤳ exercise 64

Page 246 lines −1, −3 — 17 Aug 1998

Fig. 60 ⤳ **Fig. 61**
Fig. 60 ⤳ Fig. 61

▸**Page 247** line 4 — 29 Oct 1999

$1 \le i \le N$ ⤳ $1 < i \le N$

▸**Page 250** line 22 — 21 Nov 2003

15,000000 ⤳ 15000000

▸**Page 257** in Fig. 66 — 04 Feb 2009

[delete the label `T`=leaf]

Page 257 line −16 — 04 Feb 2009

record pointed to by `LOSER`; ⤳ record stored in this external node;

Page 257 bottom two lines — 04 Feb 2009

`Q` ← `LOC(`X[0]`)`, and `RQ` ← 0. ⤳ and `Q` ← `LOC(`X[0]`)`.

Page 258 replacement for steps R2–R6 ——— 04 Feb 2009

R2. [End of run?] If RN(Q) $=$ RC, go on to step R3. (Otherwise RN(Q) $=$ RC $+ 1$ and we have just completed run number RC; any special actions required by a merging pattern for subsequent passes of the sort would be done at this point.) If RC $=$ RMAX, stop; otherwise set RC $\leftarrow$ RC $+ 1$.

R3. [Output top of tree.] (Now Q points to the "champion," and RN(Q) $=$ RC.) If RC $\neq 0$, output RECORD(Q) and set LASTKEY $\leftarrow$ KEY(Q).

R4. [Input new record.] If the input file is exhausted, set RN(Q) $\leftarrow$ RMAX $+ 1$ and go on to step R5. Otherwise set RECORD(Q) to the next record from the input file. If KEY(Q) $<$ LASTKEY (so that this new record does not belong to the current run), set RMAX $\leftarrow$ RN(Q) $\leftarrow$ RC $+ 1$.

R5. [Prepare to update.] (Now Q points to a new record.) Set T $\leftarrow$ PE(Q). (Variable T is a pointer that will move up the tree.)

R6. [Set new loser.] Set L $\leftarrow$ LOSER(T). If RN(L) $<$ RN(Q) or if RN(L) $=$ RN(Q) and KEY(L) $<$ KEY(Q), then set LOSER(T) $\leftarrow$ Q and Q $\leftarrow$ L. (Variable Q keeps track of the current winner.)

▸**Page 262** line -13 ——— 10 Jan 2003

685–687 $\leadsto$ 685–688

Page 263 the rating of exercise 11 ——— 04 Feb 2009

[*M25*] $\leadsto$ [*M20*]

Page 263 line 2 of exercise 12 ——— 04 Feb 2009

how often are interchanges made in step R6? $\leadsto$
how often does step R6 set LOSER $\leftarrow Q$?

Page 264 line 2 of exercise 23 ——— 11 May 1999

the same order that $\leadsto$ the same order as

Page 268 line 10 ——— 14 Feb 2005

the '2,2,2' on T3 in Phase 3 should be right-justified

Page 270 lines 5 and 6 ——— 21 Sep 2004

the usual Fibonacci sequence; ... by V. Schlegel $\leadsto$ the usual Fibonacci sequence, and when $p = 3$ it has been called the Tribonacci sequence. Such sequences were apparently first studied for $p > 2$ by Nārāyaṇa Paṇḍita in 1356 [see P. Singh, *Historia Mathematica* **12** (1985), 229–244], then many years later by

▸**Page 271** line 1 of step D1 ——— 06 May 1998

D[j] $- 1$ and TAPE[k] $\leftarrow j$ $\leadsto$ D[j] $\leftarrow 1$ and TAPE[j] $\leftarrow j$

Page 275 equation number in wrong font ——— 14 Nov 1999

(12) $\leadsto$ (12)

▸**Page 282** line −10 — 14 Nov 1999

$v_1 + u_0 + v_0 \leadsto v_1$

Page 283 replacement for lines −17 thru −13 — 24 May 2008

	R	$13^0 19^1$	13^1	7^0	—	31^1	$1 \times 31 = 31$	19
1	$19^1 31^0$	$13^0 19^1$	13^1	7^0	R	$31^1 52^0$	$0 \times 52 = 0$	
	$19^1 31^0$	19^1	13^1	—	52^0	R	$0 \times 52 = 0$	$\max(36, 31, 23)$
0	$19^1 31^0$	19^1	13^1	R	$52^0 82^0$	$31^1 52^0$	$0 \times 82 = 0$	
	(31^0)	(19^0)	—	82^1	(R)	$(31^0 52^0)$	$1 \times 82 = 82$	0

Page 286 line 3 of exercise 14 — 14 Nov 1999

$T_{(n(k)+1),k} \leadsto T_{(n(k)+1)k}$

▸**Page 287** the running headline — 21 Jul 2002

CASCADE $\leadsto$ POLYPHASE

Page 288 line 3 — 19 Jun 2003

National Conf. $\leadsto$ *Nat. Meeting*

Page 289 lines 8–17 — 18 Aug 2009

[this table can be compressed horizontally, so that it doesn't extend into the margin]

▸**Page 291** replacement for line 18 — 30 Apr 2008

$$D_1 \le c + d, \quad D_2 \le c, \quad D_3 = 0; \qquad D_1 \ge D_2 \ge D_3.$$

▸**Page 294** line 7 of (4) — 08 Feb 2001

$d_n = c_n - e_{n-1} \leadsto d_n = c_n - c_{n-1}$

Page 296 line −6 — 08 Apr 2003

Eqs. (8) $\leadsto$ The equations in (8)

▸**Page 299** line −18 — 16 Jan 2000

Programmer $\leadsto$ Programmers

Page 316 lines 1 and 2 — 04 Dec 2005

unless successfully contested, such a patent makes it illegal to use $\leadsto$ between 1968 and 1988, no one in the U.S.A. could legally use

▸**Page 316** the T3 entry for Phase 5 — 30 Apr 2008

$.A_1 \leadsto A_1.$

▸**Page 316** line 2 of exercise 2 — 14 Feb 2005

11 dummy runs $\leadsto$ 10 dummy runs

▸**Page 328** line 23 — 30 Apr 2008

same as in example 6. $\leadsto$ same as in example 5.

▸**Page 329** line −15 — 30 Apr 2008

$$\begin{array}{ccccc} — & — & — & A_1 A_4 & A_1 A_4 \end{array} \leadsto$$
$$\begin{array}{ccccc} A_1 & A_1 & A_1 & A_1 A_4 & A_1 A_4 \end{array}$$

▸**Page 332** line 17 ——— 28 Oct 2000

$nC(1+\rho)\omega_o\tau \leadsto NC(1+\rho)\omega_o\tau$

▸**Page 338** corrections to Chart A, the foldout illustration ——— 30 Apr 2008

[Note that this illustration consists of ten long "rows," numbered '1.' to '10.', where each row is divided into six compartments representing the activity of magnetic tapes T1, T2, T3, T4, T5, and T6, as the "time" advances from 0 to 19 minutes (indicated by vertical lines). The following corrections are described in order of increasing time, rather than in order of increasing row number.]

In row 9, between times 8.70 and 8.75: tape T4 should be textured for "reading in backward direction," as it is just before 8.70 and just after 8.75; this is the brief interval when T3 is solid black ("writing in forward direction") and T1, T2, T5 are blank.

In row 5, between times 9.5 and 10.4: tape T5 is currently marked "reading in forward direction," but it should be blank; this tape is dotted ("rewinding in backward direction") just before time 9.5; only tapes T1, T3, T4, T6 should be reading forward at this point.

In row 5, between times 10.4 and 11.2: tape T6 is currently blank; it should be dotted, to signify "rewinding in backward direction," because it is rewinding together with T2 (although T2 finishes earlier because it doesn't have to rewind as far).

▸**Page 341** line 4 ——— 30 Apr 2008

$$\frac{p_0}{1-p_1z-p_0z} \leadsto \frac{p_0}{1-p_{\geq 1}z-p_0z}$$

Page 345 lines −20, −19, and −18 ——— 17 Sep 2004

reflection shows why ... into S runs. $\leadsto$ reflection shows why this is so, if we consider the actions of a merge sort and imagine that time could run backwards: The final output is "unmerged" into subfiles, which are unmerged into others, etc.; at time zero the output has been unmerged into S runs.

Page 348 line 3 of exercise 1 ——— 10 Aug 1998

mixed radix system $\leadsto$ mixed-radix number system

▸**Page 348** line −13 ——— 18 May 1998

preform $\leadsto$ perform

▸**Page 349** line −15 ——— 30 Apr 2008

$\sqrt{24975\gamma}+\gamma^2-\gamma \leadsto \sqrt{24975\gamma+\gamma^2}-\gamma$

Page 350 in the specification of `SORT10` ——— 10 Jan 2003

Tape 0 $\leadsto$ tape 0

Page 353 line −16 ——— 25 Nov 2008

floor $i+1 \leadsto$ floor $i \pm 1$

▸**Page 356** line 1 of exercise 6 ——— 10 Jan 2003

Fig. 87 $\leadsto$ Fig. 88

Page 360 line −6 — 17 Sep 2004

operations would take $\rightsquigarrow$ operations might take

▸**Page 367** line 8 — 30 Apr 2008

minimum value in (4) $\rightsquigarrow$ minimum value in (5)

Page 369 replacement for the final three paragraphs — 17 Jul 2003

SyncSort begins by reading the first block of each run and putting these PB records into the memory pool. Each record in the memory pool is linked to its successor in the run it belongs to, except that the final record in each block has no successor as yet. The smallest of the keys in those final records determines the run that will need to replenished first, so we begin to read the second block of that run into the first buffer. Merging begins as soon as that second block has been read; by looking at its final key we can accurately forecast the next relevant block, and we can continue in the same way to prefetch exactly the right blocks to input, just before they are needed.

The three SyncSort buffers are arranged in a circle. As merging proceeds, the computer is processing data in the current buffer, while input is being read into the next buffer and output is being written from the third. The merging algorithm exchanges each record in the current buffer with the next record of output, namely the record in the memory pool that has the smallest key. The selection tree and the successor links are also updated appropriately as we make each exchange. Once the end of the current buffer is reached, we are ready to rotate the buffer circle: The reading buffer becomes current, the writing buffer is used for reading, and we begin to write from the former current buffer.

Many extensions of this basic idea are possible, depending on hardware capabilities. For example, we might use two disks, one for reading and one for writing, so that input and output and merging can all take place simultaneously. Or we might be able to overlap seek time by extending the circle to four or more buffers, as in Fig. 26 of Section 1.4.4, and deviating from the forecast input order.

Page 370 line 11 — 04 Dec 2005

D reads or writes $\rightsquigarrow$ D reads or D writes

Page 370 line 16 — 05 Feb 2009

D times faster. $\rightsquigarrow$ up to D times faster.

▸**Page 371** line −20 — 03 May 2004

$b_1 \rightsquigarrow h_1$

▸**Page 377** line 6 of exercise 7 — 28 Oct 2000

$6w_1+6w_2+7w_3+9w_4+9w_5+7w_3+4w_7+4w_8 \rightsquigarrow 6w_1+6w_2+7w_3+9w_4+9w_5+7w_6+4w_7+4w_8$

Page 383 lines −12 and −11 — 20 Sep 2005

at the beginning of each decade $\rightsquigarrow$ every ten years

Page 385 line 16 ——— 18 Feb 2002

1930 ⇝ 1929–1930

▸**Page 385** line 24 ——— 22 Aug 2007

(1938) ⇝ (1936)

▸**Page 385** line 27 ——— 22 Aug 2007

James W. Bryce, *U.S. Patent 2189024* (1940) ⇝ Ralph E. Page, *U.S. Patent 2359670* (1944)

Page 385 line −16 ——— 04 Dec 2000

stored program computer ⇝ stored-program computer

Page 386 near the bottom ——— 25 Jan 2003

line −9: Holberton ⇝ Snyder
line −1: Mrs. Holberton ⇝ Snyder

Page 387 line 22 ——— 25 Jan 2003

F. E. Holberton ⇝ Frances E. [Snyder] Holberton

Page 389 line −15 ——— 03 Jul 1999

SODA **8** (1997), 370–379 ⇝ *J. Algorithms* **31** (1999), 66–104.

Page 390 last line of exercise 3 ——— 05 Feb 2009

merging in ⇝ merging — namely, sorting when there are at most two runs — in

▸**Page 400** equation (12) ——— 23 Jun 1998

$c_1 = 1/N^\theta$ ⇝ $c = 1/N^\theta$

▸**Page 400** equation (12) ——— 20 Nov 2002

$= 0.1386$ ⇝ ≈ 0.1386

▸**Page 403** line 17 ——— 02 Jun 1998

a interesting ⇝ an interesting

Page 406 line 14 ——— 14 Nov 1999

$P_{N-1,m-1}$ ⇝ $P_{(N-1)(m-1)}$

Page 407 line 1 of exercise 17 ——— 23 May 2005

W. E. Smith ⇝ J. R. Jackson

Page 410 line −16 ——— 10 Jan 2003

$R_1 R_2 \ldots R_N$ ⇝ $R_1,\ R_2,\ \ldots,\ R_N$

Page 412 equation number in wrong font ——— 14 Nov 1999

(1) ⇝ (1)

Page 414 line −9 ——— 14 Nov 1999

number of ⇝ number of

▸**Page 416** lines 5 and 6 after the program — 14 Nov 1999

C1 is weighted more heavily than C2 ⇝ $C1$ is weighted more heavily than $C2$

Page 418 line 7 — 10 Jan 2003

$R_1 R_2 \ldots R_N$ ⇝ $R_1,\ R_2,\ \ldots,\ R_N$

▸**Page 424** line 7 of exercise 20 — 28 Aug 2004

Fibonacci search ⇝ Fibonaccian search

Page 434 line −13 — 03 May 2004

S ← LLINK(R) ⇝ `S ← LLINK(R)`

▸**Page 435** line 3 — 19 Jan 2000

about n^2 ⇝ about N^2

▸**Page 440** bottom line — 28 Jun 2004

4.55 ⇝ 4.72

Page 443 equation numbers in wrong font — 12 Dec 1999

(20) ⇝ (20)
(21) ⇝ (21)

▸**Page 445** line −10 — 24 May 2008

$C < H + 2 - P$ ⇝ $C \le H + 2 - P$

▸**Page 446** line −8 — 13 May 1998

in a several ⇝ in several

Page 446 the line after (27) — 24 May 2008

in this case ⇝ in this case, because we cannot have $C = H+2-P$ unless $P = 1$

Page 447 near the top — 03 Dec 1998

line 4: cost c ⇝ weight w
line 6: $-c$ ⇝ $-w$
line 7: it has been ⇝ an optimum tree has been
lines 8 and 9: a sequence of such transformations will make $l_k \le l_{k+1}$ ⇝ we have found an optimum tree in which $l_k = l_{k+1}$

▸**Page 447** at the end of the proof of Lemma X — 03 Dec 1998

line −4 of the proof: $\boxed{j{+}1}$ ⇝ $\boxed{j{-}1}$ (twice)
line −3 of the proof: $\boxed{i{+}1}$ ⇝ $\boxed{i{-}1}$
line −2 of the proof: $\boxed{k{+}1}$ ⇝ $\boxed{k{-}1}$

▸**Page 447** replacement for lines 22–28 — 25 Jul 2004

Suppose $l_s < l_k - 1 \le l_{s+1}$ for some s with $j \le s < k-1$. Let t be the smallest index $< k$ such that $l_t = l_k$. Then $l_i = l_k - 1$ for $s < i < t$, and $\boxed{s{+}1}$ is a left child; possibly $s+1 = t$. Furthermore $\boxed{t}$ and $\boxed{t{+}1}$ are siblings. Replace their parent by $\boxed{t{+}1}$; replace $\boxed{i}$ by $\boxed{i{+}1}$ for $s < i < t$; and replace the external node $\boxed{s}$ by an internal node whose children are $\boxed{s}$ and $\boxed{s{+}1}$. This change increases the cost by $\le q_s - q_t - q_{t+1} \le q_s - q_{k-1} - q_k$, so it is an improvement if $q_s < q_{k-1} + q_k$. Therefore, by (iii), $l_j \ge l_k - 1$.

▸**Page 447** line -9 — 25 Jul 2004

(b) $\rightsquigarrow$ (a) or (b)

▸**Page 447** replacement for line -5 — 25 Jul 2004

$$C(q'_0,\ldots,q'_{n-1}) \;\le\; C(q_0,\ldots,q_n)-(q_{k-1}+q_k). \qquad (29)$$

▸**Page 447** replacement for line -1 — 25 Jul 2004

$$\boxed{0}\quad\ldots\quad\boxed{j{-}1}\quad\boxed{k{-}1}\quad\boxed{k}\quad\boxed{j}\quad\ldots\quad\boxed{k{-}2}\quad\boxed{k{+}1}\quad\ldots\quad\boxed{n}. \qquad (30)$$

▸**Page 449** near the top — 03 Dec 1998

line 4: $\boxed{k{-}2} \rightsquigarrow \boxed{s{-}1}$
line 6: $j < i < k-1 \rightsquigarrow j < i < s$.

▸**Page 450** in Fig. 19 — 15 Sep 2004

[On the path from the root to external node G, the level numbers on internal nodes should be $(0,1,2,3,4,5)$, not $(0,1,2,2,3,4)$.]

▸**Page 451** replacement for steps G1, G2, and G3 — 18 Sep 2004

G1. [Begin phase 1.] Set $\mathtt{WT(X}_k\mathtt{)} \leftarrow w_k$ and $\mathtt{LLINK(X}_k\mathtt{)} \leftarrow \mathtt{RLINK(X}_k\mathtt{)} \leftarrow \Lambda$ for $0 \le k \le n$. Also set $\mathtt{P}_0 \leftarrow \mathtt{X}_{2n+1}$, $\mathtt{WT(P}_0\mathtt{)} \leftarrow \infty$, $\mathtt{P}_1 \leftarrow \mathtt{X}_0$, $t \leftarrow 1$, $m \leftarrow n$. Then perform step G2 for $r = 1, 2, \ldots, n$, and go to G3.

G2. [Absorb w_r.] (At this point we have the basic condition

$$\mathtt{WT(P}_{i-1}\mathtt{)} \;>\; \mathtt{WT(P}_{i+1}\mathtt{)} \qquad \text{for } 1 \le i < t; \qquad (31)$$

in other words, the weights in the working array are "2-descending.") If $\mathtt{WT(P}_{t-1}\mathtt{)} \le w_r$, set $k \leftarrow t$, perform Subroutine C below, and repeat step G2. Otherwise set $t \leftarrow t+1$ and $\mathtt{P}_t \leftarrow \mathtt{X}_r$.

G3. [Finish phase 1.] While $t > 1$, set $k \leftarrow t$ and perform Subroutine C below.

▸**Page 452** replacement for Subroutine C — 13 May 2009

Subroutine C (*Combination*). This recursive subroutine is the heart of the Garsia–Wachs algorithm. It combines two weights, shifts them left as appropriate, and maintains the 2-descending condition (31). Variables j and w are local, but variables k, m, and t are global.

C1. [Create a new node.] (At this point we have $k \ge 2$.) Set $m \leftarrow m+1$, $\mathtt{LLINK(X}_m\mathtt{)} \leftarrow \mathtt{P}_{k-1}$, $\mathtt{RLINK(X}_m\mathtt{)} \leftarrow \mathtt{P}_k$, $\mathtt{WT(X}_m\mathtt{)} \leftarrow w \leftarrow \mathtt{WT(P}_{k-1}\mathtt{)}+\mathtt{WT(P}_k\mathtt{)}$.

C2. [Shift the following nodes left.] Set $t \leftarrow t-1$, then $\mathtt{P}_j \leftarrow \mathtt{P}_{j+1}$ for $k \le j \le t$.

C3. [Shift the preceding nodes right.] Set $j \leftarrow k-2$; then while $\mathtt{WT(P}_j\mathtt{)} < w$ set $\mathtt{P}_{j+1} \leftarrow \mathtt{P}_j$ and $j \leftarrow j-1$.

C4. [Insert the new node.] Set $\mathtt{P}_{j+1} \leftarrow \mathtt{X}_m$.

C5. [Done?] If $j = 0$ or $\mathtt{WT(P}_{j-1}\mathtt{)} > w$, exit the subroutine.

C6. [Restore (31).] Set $k \leftarrow j$, $j \leftarrow t-j$, and call Subroutine C recursively. Then reset $j \leftarrow t-j$ (note that t may have changed!) and return to step C5. ▮

▸**Page 453** line 22 — 11 May 1999

It it is smaller, $\rightsquigarrow$ If it is smaller,

Page 454 line 7 before the exercises — 05 Oct 2005

See also the paper $\rightsquigarrow$ Further properties have been found by M. Karpinski, L. L. Larmore, and W. Rytter, *Theoretical Comp. Sci.* **180** (1997), 309–324. See also the paper

▸**Page 455** line 2 of exercise 11 — 11 May 1999

empirical data $\rightsquigarrow$

Page 457 line 1 of exercise 28 — 11 May 1999

a “optimum binary search” $\rightsquigarrow$ an “optimum binary search”

▸**Page 459** line 3 — 28 Jul 1999

Doklady Akademii͡a Nauk $\rightsquigarrow$ *Doklady Akademii Nauk*

Page 459 line 4 — 18 Oct 2008

Soviet Math. **3** $\rightsquigarrow$ *Soviet Math. Doklady* **3** (1962)

▸**Page 460** line 2 of Theorem A — 08 Dec 1998

1.4404 $\rightsquigarrow$ 1.4405

Page 465 line 12 — 16 Feb 2003

`K.` $\rightsquigarrow$ *K.*

Page 465 line −2 — 28 Nov 1999

$-a$, 0 or 0 $\rightsquigarrow$ $-a$, 0, or 0

Page 466 line 17 — 28 Nov 1999

$-a$, 0 or 0 $\rightsquigarrow$ $-a$, 0, or 0

▸**Page 469** lines −3 and −2 — 02 Jun 1998

$.143 + .153 + .143 + .143 = .582$ $\rightsquigarrow$ $.143 + .152 + .143 + .143 = .581$

Page 470 line −6 — 27 Dec 2000

3.3 $\rightsquigarrow$ 3

▸**Page 471** line −20 — 27 Dec 2000

Section 5.2.2 $\rightsquigarrow$ Section 6.2.2

▸**Page 473** line 5 — 28 Oct 2000

Set $\mathtt{P} = \mathtt{R}$ $\rightsquigarrow$ Set $\mathtt{P} \leftarrow \mathtt{R}$

▸**Page 475** lines 3 and 4 after the caption — 23 Oct 1998

both (although $\rightsquigarrow$ both, although

Page 476 line 9 — 22 Mar 2006

many rebalancings $\rightsquigarrow$ several rebalancings

▸**Page 477** line −5 ——— 28 Nov 1999

(1989) ⇝ (1990)

Page 478 bottom line ——— 19 Aug 1998

Lecture Notes in Comp. Sci. **1136** (1996), 91–106 ⇝ *JACM* **45** (1998), 288–323

Page 479 line 1 of exercise 7 ——— 04 Jun 2007

(N. J. A. Sloane and A. V. Aho.) ⇝ (A. V. Aho and N. J. A. Sloane.)

Page 486 lines 10 and 11 ——— 12 Nov 2008

It follows that the average ... N keys ⇝ It follows that $(p-l)/N$, the average ... N keys,

▸**Page 487** line 19 ——— 08 Dec 2000

contain m keys ⇝ contains m keys

▸**Page 489** line 27 ——— 16 Feb 2003

121–138 ⇝ 121–137

▸**Page 492** line 12 ——— 16 Feb 2003

490–500 ⇝ 490–499

▸**Page 493** at left of the table ——— 28 Nov 1999

between `I` and `J`: `Θ` ⇝ `Δ`
between `R` and `Π`: `Φ` ⇝ `Σ`

Page 496 new paragraph to follow line 23 ——— 07 Apr 2001

An interesting way to store large, growing tries in external memory was suggested by S. Y. Berkovich in *Doklady Akademii Nauk SSSR* **202** (1972), 298–299 [English translation in *Soviet Physics–Doklady* **17** (1972), 20–21].

▸**Page 500** line 9 ——— 11 Sep 1998

See n ⇝ Set n

Page 505 line −5 ——— 28 Nov 1999

node.) ⇝ node).

▸**Page 511** line 6 of exercise 41 ——— 31 Oct 2004

that can written ⇝ that can be written

Page 512 new exercise ——— 08 Mar 1999

▸**45.** [*M25*] If the seven keys of Fig. 33 are inserted in random order by the algorithm of exercise 15, what is the probability of obtaining the tree shown?

▸**Page 513** line −15 ——— 25 Feb 2000

Mecmuasi ⇝ *Mecmuası*

▸**Page 514** lines 12 and 13, in column `HER` ——— 21 Sep 2001

−1 ⇝ 1

▸**Page 514** line 10 ——— 15 Oct 1999

`K(3,3)` ⤳ `K(3:3)`

▸**Page 515** line −14 ——— 17 Jul 1998

good good spread ⤳ good spread

▸**Page 517** in 13th and 14th printings only ——— 14 Nov 2003

[The following line was missing at the bottom of the page.]
$h(2)$, ..., so the following experiment suggests itself: Starting with the line

Page 518 lines 6–8 ——— 07 Apr 2003

was first conjectured ... proof.] ⤳ was observed long ago by botanists Louis and Auguste Bravais, *Annales des Sciences Naturelles* **7** (1837), 42–110, who gave an illustration equivalent to Fig. 37 and related it to the Fibonacci sequence. See also S. Świerczkowski, *Fundamenta Math.* **46** (1958), 187–189.]

Page 519 lines −5 and −4 ——— 27 Jun 2006

Such arrays ... addition. ⤳ Such arrays make multiplication unnecessary. If M is a power of 2, we can avoid the division in (9) by substituting exclusive-or for addition; this gives a different, but equally good, hash function.

Page 521 line 10 after the illustration ——— 20 Apr 2009

so that insertions and unsuccessful searches go faster ⤳
so that unsuccessful searches — which must precede insertions — go faster

Page 523 line 2 after (13) ——— 19 Dec 1999

rA $\equiv K$. ⤳ rA $\equiv K$; rI2 $\equiv$ `LINK[`i`]` and/or R.

▸**Page 525** replacement for line 14 ——— 21 Sep 2001

$$C_N = 1 + \frac{N-1}{2M} \approx 1 + \frac{\alpha}{2} \qquad \text{(successful search).} \tag{19}$$

Page 525 line 18 ——— 02 Aug 2004

actually needed for ⤳ that is occupied by

Page 525 line −3 ——— 17 May 2001

open position ⤳ empty position

Page 527 line 3 ——— 03 Jan 2003

non-negative ⤳ nonnegative

Page 529 line 3 of (24) ——— 19 May 2005

rA $\leftarrow$ rA $\vee 1$ ⤳ rA $\leftarrow$ rA $\mid 1$

▸**Page 530** in (28) ——— 21 Sep 2001

$M+1-n$ ⤳ $M+1-N$

▸**Page 532** bottom line ——— 14 Mar 2001

`TABLE[`$(p_0 - c_1) \bmod M$`]` ⤳ `TABLE[`$(p_1 - c_1) \bmod M$`]`

▸**Page 534** line 6 ——— 17 Dec 1998

to step R2. $\rightsquigarrow$ to step R1.

Page 534 lines −4 and −3 ——— 06 Jun 2007

exactly r probes are needed $\rightsquigarrow$ any permutation of table locations needs exactly r probes

Page 537 line −1 ——— 16 Feb 2003

Eq. 1.2.6–39 $\rightsquigarrow$ Eq. 1.2.6–(39)

▸**Page 540** line 15 ——— 26 Jul 2008

C_3' $\rightsquigarrow$ C_2'

▸**Page 541** line 15 ——— 21 Sep 2001

$\binom{M}{N}$ $\rightsquigarrow$ $\binom{M}{n}$

▸**Page 547** line −12 ——— 31 Oct 2008

Boehme $\rightsquigarrow$ Boehm

Page 547 line −9 ——— 13 Feb 2003

probing. $\rightsquigarrow$ probing. [See also Derr and Luke, *JACM* **3** (1956), 303.]

▸**Page 548** line −5 ——— 07 Jul 1998

Witold Lipski $\rightsquigarrow$ Witold Litwin

Page 549 line 11 ——— 23 Apr 2009

344]. $\rightsquigarrow$ 344]; related ideas had been explored by G. D. Knott, *Proc. ACM-SIGFIDET Workshop on Data Description, Access and Control* (1971), 187–206.

▸**Page 550** line 3 ——— 20 Nov 1999

construct polynomial $\rightsquigarrow$ construct a polynomial

▸**Page 550** line 10 ——— 21 Sep 2001

coefficients p_j $\rightsquigarrow$ coefficients p_i

▸**Page 550** line −4 of exercise 8 ——— 18 Jun 1998

$\{(-1)^k q_k\theta\}$ $\rightsquigarrow$ $\{(-1)^{k+1} q_k\theta\}$

▸**Page 550** line −3 of exercise 8 ——— 24 Jul 1998

$\theta\}$ $\rightsquigarrow$ $\theta\}^+$

▸**Page 555** line 7 of exercise 55 ——— 06 Apr 2000

Eq. 2.3.4.4–(9) $\rightsquigarrow$ Eq. 2.3.4.4–(21)

▸**Page 557** line 6 of exercise 72 ——— 11 May 1999

than the expected $\rightsquigarrow$ then the expected

Page 558 new exercise ——— 26 Sep 2007

▸**78.** [*M26*] (P. Woelfel.) If $0 \le x < 2^n$, let $h_{a,b}(x) = \lfloor (ax+b)/2^k \rfloor \bmod 2^{n-k}$. Show that the set $\{h_{a,b} \mid 0 < a < 2^n,\ a \text{ odd, and } 0 \le b < 2^k\}$ is a universal family of hash functions from n-bit keys to $(n-k)$-bit keys. (These functions are particularly easy to implement on a binary computer.)

▸**Page 561** line −12 ——— 12 Dec 1999

for examples $\rightsquigarrow$ for example

▸**Page 565** line −7 ——— 07 Apr 2002

$(k \bmod l) + 1 \rightsquigarrow (l \bmod k) + 1$

Page 566 line −4 ——— 09 Sep 2010

"(`CLASS`, `MAJOR`) attribute" $\rightsquigarrow$ (`CLASS`, `MAJOR`) attribute

Page 571 first two lines of (11) ——— 20 Nov 1999

[semicolons are missing after the numeric values]

Page 571 line 15 ——— 19 May 2005

$0010001000 \vee 0000100100 \vee 0000001001 \rightsquigarrow 0010001000 \mid 0000100100 \mid 0000001001$

▸**Page 573** line −15 ——— 17 Nov 1999

IEEE/ $\rightsquigarrow$ *IEEE*

▸**Page 574** line −2 ——— 04 Aug 1998

1, 2, 5, 7, and 8 $\rightsquigarrow$ 0, 1, 4, 6, and 7

Page 575 line 18 ——— 20 Mar 2000

In 1997, A. E. Brouwer found $\rightsquigarrow$ A. E. Brouwer [*SICOMP* **28** (1999), 1970–1971] has found

Page 575 line −6 ——— 02 Jul 2001

stored three places $\rightsquigarrow$ stored thrice

▸**Page 576** near the top ——— 26 Jul 2008

line 6: 15 that use it $\rightsquigarrow$ 16 that use it
line 7: 16–15 split $\rightsquigarrow$ 16–16 split

▸**Page 577** line 3 ——— 25 Mar 2001

triples, $\rightsquigarrow$ triples.

Page 578 line 22 ——— 12 Dec 1999

Proc.ACM $\rightsquigarrow$ *Proc. ACM*

▸**Page 578** line −3 ——— 07 Apr 2005

J. Algorithms **1** $\rightsquigarrow$ *J. Algorithms* **2**

▸**Page 580** line 9 of exercise 8 ——— 17 May 2001

each of the queries $\rightsquigarrow$ each of the queries in

Page 584 quotation from Lewis Carroll ——— 13 Feb 2008

[The extra quotation marks, and the lack of a hyphen in down-stairs, are *not* erroneous. I transcribed them faithfully from page 59 of Carroll's handwritten manuscript, which was published in facsimile in 1888, of a privately circulated work that predated *Alice's Adventures in Wonderland.*]

▸**Page 584** line 6 ——— 06 Jul 1998

(1862) ⇝ (1864)

Page 584 line 11 ——— 08 Oct 2010

Annals of Mathematics ⇝ *Annals of Mathematics* (2)

▸**Page 584** line 13 ——— 24 Aug 2002

$p(1)\ldots p(n)$ and $q(1)\ldots q(n)$ ⇝ $p(1)\ldots p(N)$ and $q(1)\ldots q(N)$

▸**Page 584** line −13 ——— 06 Jul 1998

$R_{p(n)}$ ⇝ $R_{p(1)}$

▸**Page 587** last line of answer 12 ——— 03 Jan 2000

$\lceil \lg n\rceil$ ⇝ $\lceil \lg n\rceil + 1$

Page 588 line 7 ——— 17 Nov 2008

cross-reference ⇝ cross-reference card

Page 588 line 17 ——— 14 Sep 1998

`ACHTZEHNHUNDERT␣8ZWOLF␣8E` ⇝ `ACHTZEHNHUNDERTZWOLF␣8EIN`

Page 588 line 34 ——— 11 Nov 1998

`SUSSEN␣8MADE` ⇝ `LANGEN␣8TAG␣`

▸**Page 589** answer 19 ——— 09 Jul 1998

line 1: (x_i, x_j) ⇝ $\{x_i, x_j\}$
line 7: (x_i, x_i) ⇝ $\{x_i, x_i\}$
lines 7 and 8: those of exercises 18 and 19 ⇝ the method of exercise 18

Page 589 lines 10 and 11 of answer 19 ——— 24 Mar 2007

Another approach $\ldots c-x)$. ⇝ Another approach, suggested by Jiang Ling, is to sort on a key such as $(x > c/2 \Rightarrow x,\ x \le c/2 \Rightarrow c - x)$.

▸**Page 589** lines 2 and 3 of answer 20 ——— 19 May 2005

'$\wedge$' ⇝ '&' (three changes)

▸**Page 589** line 2 of answer 21 ——— 05 Mar 1999

`ERS`, ⇝ `-ERS`,

Page 589 bottom line ——— 28 Jan 2001

Dudeney, ⇝ Dudeney, *Strand* **65** (1923), 208, 312, and his

Page 590 line 11 ——— 04 May 2002

m is the computer word size. Sorting the file $\bigl(f(\alpha), \alpha\bigr)$ will ⇝ m is, say, $2^{\lfloor 2\lg N\rfloor}$ when there are N words. Sorting the file $\bigl(f(\alpha), \alpha\bigr)$ with two passes of Algorithm 5.2.5R will bring anagrams

Page 590 last line of answer 22 — 24 Aug 2002

142.] $\rightsquigarrow$ 142].

▸**Page 592** line 6 of answer 7 — 28 Oct 2000

Rodriguez $\rightsquigarrow$ Rodrigues

Page 592 lines 6 and 7 of answer 7 — 19 Dec 2001

240; the C inversion table appears in $\rightsquigarrow$ 240. The C inversion table was used by Rothe in 1800; see also

▸**Page 593** answer 13 — 02 Jun 2007

line 1: b_{m-1}, b_{m+1} $\rightsquigarrow$ b_{n-m}, b_{n-m+2}
line 2: b_m $\rightsquigarrow$ b_{n-m+1}

Page 594 line 1 — 11 Dec 2002

Euler's $\rightsquigarrow$ *Comptes Rendus Acad. Sci.* **92** (Paris, 1881), 448–450. Euler's

Page 594 line 1 of answer 20 — 22 Aug 2002

See $\rightsquigarrow$ See J. J. Sylvester, *Amer. J. Math.* **5** (1882), 251–330, **6** (1883), 334–336, §57–§68;

▸**Page 594** line 1 of answer 20 — 07 Jan 2000

Zolnowski $\rightsquigarrow$ Zolnowsky

▸**Page 595** line 3 — 18 May 2000

$(uv)^{\binom{k}{2}}(-u^{-n}v^{1-n})^k \rightsquigarrow (uv)^{\binom{j}{2}}(-u^{-n}v^{1-n})^j$

▸**Page 595** line 6 of answer 23 — 29 Jun 1999

$(q_n^{h_1+k_1}p_n) \rightsquigarrow (q_n^{h_1+k_n}p_n)$

▸**Page 596** line 3 of answer 27 — 03 Jan 2000

$$\sum_n \frac{H_n(w,z)}{(1-z)\ldots(1-z^n)} \rightsquigarrow \frac{H_n(w,z)}{(1-z)\ldots(1-z^n)}$$

Page 597 replacement for last paragraph of answer 28 — 23 Nov 2002

The average total displacement of a random permutation is $(n^2-1)/3$; see exercise 5.2.1–7. The generating function for total displacement does not appear to have a simple form. *References:* C. Spearman, *British J. Psychology* **2** (1906), 89–108; P. Diaconis and R. L. Graham, *J. Royal Stat. Soc.* **B39** (1977), 262–268.

▸**Page 598** line 2 of answer 11 — 01 Jan 2006

if and only if ... and that $\rightsquigarrow$ if and only if we have $\sigma_{p(1)} \intercal \cdots \intercal \sigma_{p(t)} = \sigma_1 \intercal \cdots \intercal \sigma_t$ and $p(i) < p(j)$ whenever $\sigma_{p(i)} = \sigma_{p(j)}$ for $i < j$. We also want to show that,

▸**Page 599** answer 14 — 01 Nov 2002

line 11: $(l \cdot a,\, m \cdot b,\, n \cdot c) \rightsquigarrow \{l \cdot a,\, m \cdot b,\, n \cdot c\}$
line 14: $(n_1 \cdot x_1, \ldots, n_m \cdot x_m) \rightsquigarrow \{n_1 \cdot x_1, \ldots, n_m \cdot x_m\}$

Page 599 line 1 of answer 15 — 01 Jan 2006

Theorem 2.3.4.2D $\rightsquigarrow$ Theorem 2.3.4.2D and Lemma 2.3.4.2E

▸**Page 602** last line of answer 2 — 01 Jan 2006

$m = n - q$ and $n - q - 1 \leadsto q = n - m$ and $q = n - m + 1$.

▸**Page 603** line 1 of answer 11 — 09 Aug 1999

$\sum_{t_1 \ge 1, \ldots, t_k \ge 1} \leadsto \sum_{t_1 \ge 1, \ldots, t_{k-1} \ge 1}$

▸**Page 603** replacements for lines 6, 7, and 9 of answer 11 — 01 Jan 2006

$$E_1(n) = 1/(n+1)! - 1/n!\,; \qquad E_2(n) = [n>0]/(n+1)!\,;$$

$$E_k(n) = (-1)^k \sum_{m \ge 0} \binom{m}{k-3} \frac{[n>0]}{(n+2+m)!}, \qquad k \ge 3.$$

$$E(z,x) = \sum_{n,k} E_{k+1}(n) z^n x^k = \frac{ez^2x^2 - e^x(1-x+zx)(z+x-1) - e^{z+x}(1-z)^2(1-x)^2}{e^x z(z+x-1)(1-x)^2}.$$

Page 604 answer 16 — 14 Aug 2002

[change the notation from $\left|{n \atop k}\right|$ to $\wr{n \atop k}\wr$, because I'm now using the former notation for partition numbers in Volume 4]

Page 605 line 9 — 22 Aug 2002

$\langle\!\langle {n \atop k} \rangle\!\rangle \leadsto \wr{n \atop k}\wr$

Page 605 near the top — 22 Aug 2002

line 8: *Paris* **81** (1875) $\leadsto$ **81** (Paris, 1875)
line 9: *Paris* **97** (1883) $\leadsto$ **97** (Paris, 1883)
line 10: (Paris: 1884) $\leadsto$ (1884)

▸**Page 605** line −7 of answer 16 — 01 Jan 2006

$g_n(-1) = 0 \leadsto G_n(-1) = 0$

Page 605 replacement for lines 4 and 5 of answer 19 — 25 Nov 2003

[*Duke Math. J.* **13** (1946), 259–268. Sections 2.3 and 2.4 of Richard Stanley's *Enumerative Combinatorics* **1** (1986) discuss rook placement in general.]

▸**Page 606** line −6 of answer 23 — 01 Jan 2006

$(u^m + (1-u)^m + |u-x|^m)/m! \leadsto (u^m + (1-u)^m - |u-x|^m)/m!$

Page 606 last line of answer 25 — 12 Feb 2005

722]. $\leadsto$ 722]; see also W. Meyer and R. von Randow, *Math. Annalen* **193** (1971), 315–321.

Page 607 new answer — 26 May 2000

28. The poles of $L(z)$ are the values of $T(1/e)$, where $T(z)$ is the (multivalued) tree function defined by $T(z) = ze^{T(z)}$. Thus for $m > 0$ we have the convergent series

$$z_m = -\sigma_m + \sum_{n \ge 0} \frac{1}{\sigma_m^n} \sum_k (-1)^k \left[{n \atop k}\right] \frac{(\ln \sigma_m)^{n+1-k}}{(n+1-k)!}, \qquad \sigma_m = -1 - (2m+1)\pi i$$

[Corless, Gonnet, Hare, Jeffrey, and Knuth, *Advances in Computational Mathematics* **5** (1996), 329–359, formula (4.18)]; in particular, we have $z_m = (2m+\frac{1}{2})\pi i + \ln(2\pi e m) + \left(\frac{1}{4} - \frac{i}{2\pi}\ln(2\pi e m)\right)/m + O\big((\log m)^2/m^2\big)$.

Let $P(z) = \sum_{m=0}^{\infty}\bigl(z/(z-z_m) + z/(z-\bar{z}_m)\bigr)$. It follows that $P(x) - P(-x) = \sum_{m=0}^{\infty} 4\Re\bigl(xz_m/(x^2-z_m^2)\bigr) = \sum_{m=1}^{\infty} O\bigl((x\log m)/(x^2+m^2)\bigr) = \sum_{m=1}^{x} O\bigl((x\log x)/x^2\bigr) + \sum_{m=x+1}^{\infty} O\bigl((x\log m)/m^2\bigr) = O(\log x)$ for $x > 1$. But we know that $L(x) + P(x) = cx$ for some c; hence $2cx = L(x) - L(-x) + O(\log x)$, and by letting $x \to \infty$ in (25) we find $c = -1/2$. Hence $L_1 = \sum_{m=0}^{\infty} 2r_m^{-1}\cos\theta_m - 1/2$. (This result is due to Svante Janson.)

Page 607 another new answer ——— 29 Jan 2005

29. (a) If $a_1 \ldots a_n$ has $2k$ alternating runs and k peaks, $(n+1-a_1)\ldots(n+1-a_n)$ has $k-1$ peaks. (b,c) See L. W. Shapiro, W.-J. Woan, and S. Getu, *SIAM J. Algebraic and Discrete Methods* **4** (1983), 459–466.

Page 609 lines 13 and 14 ——— 18 Aug 2002

[Change the notation from 'a_m' to the more modern 'e_m'.]

▶**Page 610** answer 23 ——— 01 Jan 2006

line 4: row k $\rightsquigarrow$ row k from the bottom

line 8: $\displaystyle\sum_{n\ge1} \rightsquigarrow \sum_{n\ge0}$

Page 610 and 611, in answer 23 ——— 20 Aug 2007

The coefficients A_{2n} are therefore ... tangent numbers and Euler numbers $\rightsquigarrow$
The coefficients E_n are called *Euler numbers*; with odd index, E_{2n-1} is the tangent number $T_{2n-1} = (-1)^{n-1}4^n(4^n-1)B_{2n}/(2n)$. Tables of these numbers appear in *Math. Comp.* **21** (1967), 663–688; the sequence begins $(E_0, E_1, E_2, \ldots) = (1, 1, 1, 2, 5, 16, 61, 272, 1385, 7936, \ldots)$. The easiest way to compute Euler numbers

Page 611 last two lines of answer 23 ——— 13 Jul 2007

[A. J. Kempner ... 349] $\rightsquigarrow$ [L. Seidel, *Sitzungsberichte math.-phys. Classe Akademie Wissen. München* **7** (1877), 157–187]

Page 611 lines 3–5 of answer 28 ——— 23 Mar 2000

[M. Talagrand ... $\Theta(n^{1/6})$.] $\rightsquigarrow$
[J. Baik, P. Deift, and K. Johansson, *J. Amer. Math. Soc.* **12** (1999), 1119–1178, showed that the standard deviation is $\Theta(n^{1/6})$; moreover, the probability that the length is less than $2\sqrt{n} + tn^{1/6}$ approaches $\exp\bigl(-\int_t^{\infty}(x-t)u^2(x)\,dx\bigr)$, where $u''(x) = 2u^3(x) + xu(x)$ and $u(x)$ is asymptotic to the Airy function $\mathrm{Ai}(x)$ as $x \to \infty$.]

▶**Page 611** line 3 of answer 29 ——— 28 Oct 2000

$\le \sqrt{n}/e.$ $\rightsquigarrow$ $\le \sqrt{n}/e.)$

▶**Page 612** replacement for line -2 of answer 33 ——— 01 Jan 2006

$\ldots, a+1, a, k-1, \ldots, 1, 0)/\Delta(k+l, \ldots, 1, 0)$. The value of $\Delta(k, \ldots, 1, 0) = k! \ldots 2!\,1!$

▶**Page 612** replacement for line 5 of answer 34 ——— 01 Jan 2006

the shape. If $y_{a+1} = y_a$, the cell (x_a, y_1) has a hook of length a; otherwise (x_{a+b}, y_{a+1})

▶**Page 612** line 12 of answer 35 ——— 01 Jan 2006

H4. [Move down or left.] $\rightsquigarrow$

H4. [Increase p.] Increase p_{lk} by 1.

H5. [Move down or left.]

Page 613 line 4 of answer 37 ——— 19 Aug 2002

211 $\rightsquigarrow$ **A211**

▸**Page 613** line 1 of answer 39 ——— 01 Jan 2006

original permutations $\rightsquigarrow$ original permutation

▸**Page 614** line 5 of answer 41 ——— 01 Jan 2006

$n+$ (length $\rightsquigarrow$ $n-$ (length

Page 614 line −8 ——— 14 Aug 2001

Lecture Notes in Comp. Sci. **807** (1994), 307–325 $\rightsquigarrow$ *Algorithmica* **13** (1995), 180–210

Page 615 replacement for lines 2–4 ——— 12 Dec 2008

reversals was developed by S. Hannenhalli and P. Pevzner, *JACM* **46** (1999), 1–27. Improvements that solve the problem in $O(n^{1.5}\sqrt{\log n})$ time were subsequently found by H. Kaplan and E. Verbin, *J. Comp. Syst. Sci.* **70** (2005), 321–341; E. Tannier, A. Bergeron, and M.-F. Sagot, *Discrete Applied Math.* **155** (2007), 881–888.]

Page 619 new sentence for end of answer 7 ——— 23 Nov 2002

Incidentally, the *variance* of the stated sum can be shown to equal $[n>1](2n^2+7)(n+1)/45$.

▸**Page 620** throughout answer 10 ——— 08 Mar 2001

change the local symbols `3F`, `3H`, `8F`, `4H`, `5H`, `6H`, `7F`, `5B`, `7H`, respectively to `0F`, `0H`, `7F`, `3H`, `4H`, `5H`, `6F`, `4B`, `6H`.

Page 620 line 12 ——— 02 Aug 1998

$NT-S-C \rightsquigarrow NT-S-C$ ▮

▸**Page 620** line 2 of answer 15 ——— 01 Nov 2002

$g_{n-1} \rightsquigarrow g_{n-1}(z)$

Page 623 replacement for lines 7 and 8 of answer 29 ——— 08 Dec 2003

On the other hand, Marcin Ciura's experiments [*Lecture Notes in Comp. Sci.* **2138** (2001), 106–117] indicate that the minimum 7-pass B_{ave} (≈ 6879) is obtained with increments 229 96 41 19 10 4 1, while the sequence 737 176 69 27 10 4 1 yields the smallest total sorting time ($\approx 125077u$).

Page 624 line 22 of answer 31 ——— 15 Aug 1998

[center the 'T' in the frequency column]

▸**Page 624** line 23 of answer 31 ——— 15 Aug 1998

is the $\rightsquigarrow$ is

Page 625 line −2 of answer 33 ——— 02 Aug 1998

$N-1 \rightsquigarrow N-1$ ▮

Page 625 bottom line ——— 02 Aug 1998

$M \rightsquigarrow M$ ▮

▸**Page 626** last line of answer 36 ——— 15 Aug 1998

slow!) $\rightsquigarrow$ slow!

▸**Page 626** replacement for line 13 of answer 37 ——— 22 May 2000

$$\sum_{N\geq 0} g''_{NM}(1)\frac{M^N w^N}{N!} = M(M-1)e^{(M-2)w}\left(\frac{w^2}{4}e^w\right)^2 + Me^{(M-1)w}\left(\frac{w^4}{16}+\frac{5w^3}{18}\right)e^w.$$

[Also change $g'_{MN}(1) \rightsquigarrow g'_{NM}(1)$ on line 11.]

▸**Page 626** line 2 of answer 38 ——— 22 May 2000

converges to ⤳ is asymptotic to

▸**Page 627** answer 41 ——— 28 Oct 2000

line 1: We have ⤳ (a) We have
line 2: $\rho^{k+1}/(k+1) - \rho^k/k \rightsquigarrow (\rho^{k+1}/(k+1) - \rho^k/k)/\ln\rho$
line 6: $(k-1)^2 \rightsquigarrow (k-2)^2$
line 8: $(\log\log N)^{-3} \rightsquigarrow (\log\log N)^{-2}$

▸**Page 627** line 10 of answer 42 ——— 22 May 2000

$N/gh \rightsquigarrow N/gh+1$

Page 627 bottom line ——— 11 Apr 2001

307.] ⤳ 307.] However, with a decent sequence of increments the inner loop is not performed often enough to make this change desirable.

Page 628 line 1 of answer 2 ——— 16 Aug 2001

Philos. Mag. **34** ⤳ *Philosophical Mag.* (3) **34**

▸**Page 629** line 9 of answer 12 ——— 22 Aug 2000

05 ⤳ *06*

Page 630 line 2 ——— 02 Aug 1998

$p \neq 0.$ ⤳ $p \neq 0.$ ▮

▸**Page 632** last line of answer 23 ——— 05 Jun 2000

the form (20) ⤳ the form (19)

▸**Page 634** line −7 of answer 32 ——— 04 Oct 2005

$H_j + H_{n+1-k} \rightsquigarrow H_k + H_{n+1-j}$

▸**Page 635** last line of answer 38 ——— 14 Sep 2000

$X_N = \frac{1}{2}(A_N - L_N) \rightsquigarrow X_N = \frac{1}{2}A_N$

▸**Page 636** last line of answer 41 ——— 15 Aug 1999

Chapter 11 ⤳ Chapter 14

Page 636 lines 7 and 8 of answer 44 ——— 15 Aug 1999

[insert a bit of space between these lines]

▸**Page 636** line 5 of answer 48 ——— 05 Jun 2000

$-\,\delta_0(n) \rightsquigarrow -\delta_0(n) + O(n^{-100})$

▸**Page 636** replacement for answer 49 ——— 05 Jun 2000

49. The right-hand side of Eq. (40) can be improved to the estimate $e^{-x}\bigl(1-\frac{1}{2}x^2/n + O((x^3{+}x^4)n^{-2})\bigr)$. The effect is to subtract half the sum in exercise 47, replacing $O(1)$ in (50) by $2 - \frac{1}{2}\bigl(1/\ln 2 + \delta_1(n)\bigr) + O(n^{-1})$. (The "2" comes from the "$2/n$" in (46).)

▸**Page 637** lines 3 and 4 ——— 05 Jun 2000

.0000001725, 00041227, ..., .341 ⇝
.000000172501, .000041227, .0002963, .0008501433, .0062704, .06797, .1525, .348

▸**Page 637** line 2 of answer 53 ——— 15 Aug 1999

$(p^k q^{n-k} + q^k p^{n-k})x_n$ ⇝ $(p^k q^{n-k} + q^k p^{n-k})x_k$

Page 638 last line of answer 54 ——— 15 Aug 1999

Über ⇝ *über*

▸**Page 638** line −10 of answer 55 ——— 04 Aug 1999

R_{i+1} ⇝ R_{l+1}

▸**Page 638** last two lines of answer 55 ——— 05 Aug 1999

does not look at ...fast. ⇝ does not look at K_{N+1}, but it still might examine K_0 in step Q9.

Page 641 line 3 of answer 15 ——— 22 Apr 2006

Step 1. ⇝ **P1.** [Initialize.]

▸**Page 641** line −2 ——— 01 Jan 2002

the smallest prime divisor ⇝ a prime divisor

▸**Page 642** line 1 ——— 22 Apr 2006

the queue element ⇝ a queue element

Page 642 opening lines ——— 22 Apr 2006

line 1: **Step 2.** ⇝ **P2.** [Advance q.]
line 4: **Step 3.** ⇝ **P3.** [Check for prime n.]
line 6: **Step 4.** ⇝ **P4.** [Check for prime $\sqrt{n}$.]
line 9: **Step 5.** ⇝ **P5.** [Advance n.]

▸**Page 642** line 9 ——— 02 Aug 1998

return to (b). ⇝ return to P2. ▮

▸**Page 642** line 16 ——— 06 Aug 2001

$O(N \log N)$ ⇝ $O(N \log N \log\log N)$

Page 642 last line of answer 15 ——— 18 Oct 2006

Section 7.1 ⇝ Section 7.1.3

Page 642 answer 16 ——— 22 Apr 2006

line 1: **Step 1.** ⇝ **I1.** [Make a new leaf j.]
line 2: **Step 2.** ⇝ **I2.** [Find parent of j.]
line 3: **Step 3.** ⇝ **I3.** [Done?]
line 4: **Step 4.** ⇝ **I4.** [Sift and move j up.]
line 4: return to step 2. ⇝ return to I2. ▮

▸**Page 643** line 1 ——— 22 Apr 2006

$13N \lg N + O(N)$ ⇝ $(13N \lg N + O(N))u$

▸**Page 643** line 3 of answer 21 ——— 23 Dec 2002

$\sum_{0\le q<k} 2^q \rightsquigarrow \sum_{0\le q\le k} 2^q$

▸**Page 643** line −2 of answer 23 ——— 14 Sep 2000

$\lfloor N/2\rfloor \rightsquigarrow \lceil N/2\rceil - 1$

▸**Page 643** last line of answer 25 ——— 14 Sep 2000

$(2n-3) \rightsquigarrow (2n-1)$

▸**Page 644** line −7 ——— 23 Dec 2002

$$\sum_{j=0}^{t-1} \rightsquigarrow \sum_{j=0}^{t-2}$$

▸**Page 644** line −2 ——— 15 Aug 1999

$h_n^{-1} \rightsquigarrow h_N^{-1}$

▸**Page 648** line 5 of answer 15 ——— 04 Dec 2005

twelve $\rightsquigarrow$ eighteen

▸**Page 648** line 6 of answer 15 ——— 17 Jul 2003

$(p,q,r) \rightsquigarrow (p,q,s)$

Page 648 last line of answer 15 ——— 17 Feb 2006

$8N\lg N + O(N) \rightsquigarrow (8N\lg N + O(N))u$

Page 650 line 12 of answer 5 ——— 02 Aug 1998

`DEC1  1` $\rightsquigarrow$ `DEC1  1`

▸**Page 652** replacement for line 4 of answer 18 ——— 05 Oct 1999

$\frac{1}{2}\sum_{k=0}^{CN-1}\sum_j \binom{N}{j} p_k^j (1-p_k)^{N-j}\binom{j}{2} = \frac{1}{2}\sum_{k=0}^{CN-1}\binom{N}{2}p_k^2 \le \frac{N-1}{4}\sum_{k=0}^{CN-1} p_k B/C$, because $p_k \le B/CN$.

▸**Page 653** line 3 of answer 3 ——— 05 Oct 1999

Eq. 1.2.9–(10).) $\rightsquigarrow$ Eq. 1.2.9–(10).

Page 653 line 13 of answer 3 ——— 16 Aug 2001

Phil. Mag. **18** $\rightsquigarrow$ *Phil. Mag.* (4) **18**

Page 655 line 2 of answer 7 ——— 05 Oct 1999

$6 \rightsquigarrow 2\cdot 3$

▸**Page 657** line −5 ——— 05 Oct 1999

noninteger, $\rightsquigarrow$ noninteger.

Page 658 answer 35 ——— 20 Aug 2002

[delete this answer, as the exercise has changed]

▸**Page 658** answer 1 ——— 09 Jul 1998

$S(n)+S(n) \rightsquigarrow S(m)+S(n)$

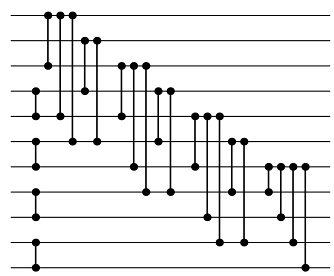
Page 664 lines 2–4 of answer 23 ——— 09 Feb 2002

D. Dor . . . proved $\rightsquigarrow$ D. Dor and U. Zwick have shown that the actual lower limit is strictly greater than 2, while the upper limit is less than 2.942 [*SICOMP* **28** (1999), 1722–1758; *SIAM J. Disc. Math.* **14** (2001), 312–325]. They also have proved

Page 666 new sentence for answer 7 ——— 16 Jun 2002

(Notice that the modified network has delay 8.)

Page 668 new answer ——— 12 Sep 2003



20. (a) First note that $\hat{V}_3(n) \ge \hat{V}_3(n-1) + 2$ when $n \ge 4$: By symmetry the first comparator may be assumed to be $[1:n]$; after this must come a network to select the third largest of $\langle x_2, x_3, \ldots, x_n\rangle$, and another comparator touching line 1. On the other hand, $\hat{V}_3(5) \le 7$, since four comparators find the min and max of $\{x_1, x_2, x_3, x_4\}$, then we sort the other three.

(b) A subtle construction by M. W. Green, shown for $n = 11$, does the job. (Equality probably holds.)

Page 669 last three lines of answer 31 ——— 01 Nov 2005

No simple . . . $\rightsquigarrow$ The asymptotic formula $\delta_{2m} = \exp\bigl(\binom{2m-1}{m}\ln 2 + \binom{2m}{m+1}/2^{m+1} + \frac{1}{16}(m-1)\sqrt{m/\pi} + O(m^{-1/2})\bigr)$ has been established by A. D. Korshunov and A. A. Sapozhenko, with a similar formula for δ_{2m+1}; see *Russian Math. Surveys* **58** (2003), 929–1001, Theorem 1.8.

Page 669 notation change in answer 32 ——— 08 Aug 2005

line 1: $\theta \le \psi \rightsquigarrow \theta \subseteq \psi$
line 3: "$\le$" $\rightsquigarrow$ "$\subseteq$"

▸**Page 669** line 4 of answer 32 ——— 05 Oct 1999

$G_T \rightsquigarrow G_t$

Page 669 last line of answer 32 ——— 08 Aug 2005

$z[(x_1 \ldots x_t)_2] \rightsquigarrow z_{(x_1 \ldots x_t)_2}$

▸**Page 669** answer 35 ——— 05 Oct 1999

line 4: and$[n-1{:}n]$ $\rightsquigarrow$ and $[n-1{:}n]$
line 5: $(k-1)D_{k-1} \rightsquigarrow (k-1)D_{k+1}$

Page 670 last line of answer 37 ——— 28 Jan 2001

Amusements $\rightsquigarrow$ *Strand* **46** (1913), 352, 472; *Amusements*

Page 670 line −11 ——— 22 Aug 2002

Paris (I) **295** (1982) $\rightsquigarrow$ (I) **295** (Paris, 1982)

▸**Page 671** line 3 of answer 40 ——— 27 Oct 1998

$t = 4n \rightsquigarrow t = 4n + \sqrt{n}\,\ln n$

▸**Page 671** replacement for lines 4–7 of answer 40 ——— 28 Oct 1998

Experiments show that the expected time to reach *any* primitive sorting network — not necessarily the bubble sort — is very nearly $2n^2$. Curiously, R. P. Stanley and S. V. Fomin have proved that if the comparators $[i_k : i_k{+}1]$ are chosen nonuniformly in such a way that $i_k = j$ occurs with probability $j/\binom{n}{2}$, the corresponding expected time comes to exactly $\binom{n}{2} H_{\binom{n}{2}}$.

▸**Page 671** line 1 of answer 45 ——— 03 Feb 2005

at most 2^t ⇝ at most 2^l.

▸**Page 671** lines 7 and 8 of answer 46 ——— 03 Feb 2005

either $y_{k+1} \le \cdots \le y_n$ or ... at least $\min(n{-}k{+}1, k) \ge \min(n{-}l{+}1, l{+}m)$
⇝ either $y_{k+1} \le \cdots \le y_n$ and $k \le l$ or $y_1 \le \cdots \le y_{k-1}$ and $k \ge l + m$; so at least $\min(n{-}l{+}1, l{+}m)$

▸**Page 671** line 4 of answer 49 ——— 05 Oct 1999

$\{0, 0, 0, 1, 1, 1\})$ ⇝ $\{0, 0, 0, 1, 1, 1\}$

Page 672 line −11 ——— 11 May 1999

consists of two ⇝ consist of two

Page 676 lines 1 and 2 of answer 66 ——— 14 Feb 2005

no pair sequence ... 10 comparisons. ⇝
every pair sequence beginning with $(1, 2)(2, 3)(3, 4)(4, 5)(1, 5)$ will avoid at least one subsequent comparison.

Page 676 replacement for answer 67 ——— 14 Feb 2005

67. Suppose $c_i = j$ for exactly t_j values of i. For the restricted case we need to prove that $\sum_j t_j/(2 + j)$ is minimized when $(t_0, t_1, \ldots, t_{N-2}) = (N{-}1, \ldots, 2, 1)$. Gil Kalai has shown that the achievable vectors $(t_0, t_1, \ldots, t_{N-2})$ are always lexicographically $\ge (N{-}1, \ldots, 2, 1)$; see *Graphs and Combinatorics* **1** (1985), 65–79.

Page 677 answer 8 ——— 04 Feb 2009

line 3: `RQ` ⇝ `RC`
line 4: `RQ` ⇝ `RC`
line 5: step R3. ⇝ step R3. (Further optimization is possible because of answer 10.)

Page 677 line 3 of answer 9 ——— 04 Feb 2009

`RN(T)` = `RQ` ⇝ `RN(L)` = `RN(Q)`

Page 677 line 3 of answer 9 ——— 03 Feb 2010

`KEY(LOSER(T))` versus `KEY(Q)` ⇝ `KEY(L)` versus `KEY(Q)`

Page 677 replacement for answer 10 ——— 04 Feb 2009

10. Let $\cdot j \equiv$ `LOC(`$X[j]$`)`, and suppose we add the unnecessary assignment ‘`LOSER(`$\cdot 0$`)` $\leftarrow$ `Q`’ at the beginning of step R3. The mechanism of Algorithm R ensures that the following conditions are true just after we’ve done that assignment: The values of `LOSER(`$\cdot 0$`)`, ..., `LOSER(`$\cdot(P-1)$`)` are a permutation of $\{\cdot 0, \cdot 1, \ldots, \cdot(P-1)\}$; and there exists a permutation of the pointers $\{$`LOSER(`$\cdot j$`)` $\mid$ `RN(LOSER(`$\cdot j$`))` $= 0\}$ that corresponds to an actual tournament. In other words, when `RN(·j)` is zero, the value of `KEY(·j)` is irrelevant; we may permute such "winners" among themselves. After P iterations all `RN(·j)` will be nonzero, so the entire tree will be consistent. (The answer to the hint is "yes.")

David P. Kanter observes that we can go directly from R6 to R4 as soon as `RN(Q)` $=$ 0, thereby avoiding all comparisons that involve uninitialized keys when $N \ge P$.

Page 678 answer 15 ——— 05 Feb 2009

but only ... exhausted. $\rightsquigarrow$ whenever all records in memory belong to the same run at the moment the input is exhausted (for example, in a one-pass sort).

▸**Page 681** line 4 of answer 6 ——— 10 Jan 2003

$\alpha p(\alpha^{-1} q''(\alpha^{-1})/q'(\alpha^{-1})^3$, $\rightsquigarrow$ $\alpha p(\alpha^{-1}) q''(\alpha^{-1})/q'(\alpha^{-1})^3$,

Page 681 lines -3 and -2 ——— 29 Nov 2009

The case $p = 2$ ... in exercise $\rightsquigarrow$ This number system was implicitly known in 14th-century India (see Section 7.2.1.7). We have considered the case $p = 2$ in exercise

Page 685 line 1 of answer 5 ——— 10 Jan 2003

k pass $\rightsquigarrow$ kth pass

▸**Page 688** line -9 ——— 10 Jan 2003

$3c_n + 2d_n + c_n$ $\rightsquigarrow$ $3c_n + 2d_n + e_n$

Page 690 line 5 of answer 23 ——— 14 Nov 1999

$u^{(1)}, \ldots.$ $\rightsquigarrow$ $u^{(1)}, \ldots, u^{(q-1)}$.

Page 692 near the top ——— 10 Jan 2003

line 1: $D[p]$ $\rightsquigarrow$ `D[`p`]`
line 2: $D[p]$ $\rightsquigarrow$ `D[`p`]`
line 2: $D[q]$ $\rightsquigarrow$ `D[`q`]`

Page 692 answer 5.4.6–5 ——— 10 Jan 2003

line 4: $C[i]$ $\rightsquigarrow$ `C[`i`]`
line 5: $C[j]$ $\rightsquigarrow$ `C[`j`]`

▸**Page 692** answer 7 ——— 04 Jun 2008

line 1: $A_0D_0(A_1D_1)^2$ $\rightsquigarrow$ $A_0D_0A_1D_1A_0D_0(A_1D_1)^3$
line 1: $D_1(A_1D_1)^4$ $\rightsquigarrow$ $D_1(A_1D_1)^2$
line 6: $D_{13}A_4D_4A_0D_0A_{10}$ $\rightsquigarrow$ $D_{13}A_4D_4A_4D_4A_0D_0A_{10}$

▸**Page 696** line 7 of answer 2 ——— 15 Mar 2001

$k+1 = q_{i(q+1)}$ $\rightsquigarrow$ $k+1 = a_{i(q+1)}$

▸**Page 697** line 3 of answer 9 ——— 16 Mar 2001

$D(\tau) = dE(\tau)$, $E(\tau) = tn + dr$ $\rightsquigarrow$ $D(\mathcal{T}) = dE(\mathcal{T})$, $E(\mathcal{T}) = tn + dr$

Page 703 new material after line 12 of answer 11 ——— 21 Jul 2010

In fact, this distribution had already been obtained by M. L. Tsetlin in his Ph.D. thesis at Moscow University in 1964, and published in Chapter 1 of his Russian book *Studies in Automata Theory and Simulation of Biological Systems* (1969).

Page 704 line 2 of answer 17 ——— 23 May 2005

Of course $\rightsquigarrow$ Management Science Research Report 43, UCLA (1955). Of course

▶**Page 704** answer 18 ——— 14 Nov 1999

first line: $q_j < r_1 \rightsquigarrow q_j < r_j$
last line: 10).] $\rightsquigarrow$ 10.]

▶**Page 704** replacement for line 2 of answer 19 ——— 09 Aug 2009

$$\sum_{i,j} p_i p_j\, d(i,j) = \tfrac{1}{2} \sum_{i,j} p_i p_j \bigl(d(i,j) + d(j,i)\bigr)[i \neq j] = \tfrac{1}{2}(1 - p_1^2 - \cdots - p_N^2)c.$$

▶**Page 708** line 3 of answer 2 ——— 12 Jan 2006

"`RTAG(P)` $\neq 0$" $\rightsquigarrow$ "`RTAG(P)` $= 0$"

▶**Page 711** line 8 of answer 32 ——— 20 Apr 2001

$\max \sum_{k=0}^{n} q_k \rightsquigarrow \max_{k=0}^{n} q_k$

▶**Page 712** lines 5 and 10 of answer 40 ——— 03 Dec 1998

$q_{k-2}(l_{k-2} - l_{k-4}) \rightsquigarrow q_{k-2}(l_{k-4} - l_{k-2})$

▶**Page 712** replacement for answer 42 ——— 18 Sep 2004

42. Let $q_j = \texttt{WT(P}_j\texttt{)}$. Steps C1–C4, which move $q_{k-1} + q_k$ into position between q_{j-1} and q_j, can spoil (31) only at the point $i = j - 1$.

▶**Page 713** in answer 44 ——— 15 Sep 2004

line 4: $\texttt{LLINK(X}_j\texttt{)} \leftarrow t \rightsquigarrow \texttt{LLINK(X}_j\texttt{)} \leftarrow \texttt{X}_t$
line 6: $\texttt{RLINK(X}_j\texttt{)} \leftarrow t \rightsquigarrow \texttt{RLINK(X}_j\texttt{)} \leftarrow \texttt{X}_t$

Page 713 lines 6–8 of answer 49 ——— 25 Apr 2010

Luc Devroye and Bruce Reed . . . , where $\rightsquigarrow$ Bruce Reed [*JACM* **50** (2003), 306–332] and Michael Drmota [*JACM* **50** (2003), 333–374], who proved that the average height is $\alpha \ln n - (3\alpha \ln\ln n)/(2\alpha - 2) + O(1)$ and the variance is $O(1)$, where

▶**Page 715** line 2 of answer 9 ——— 29 Jul 1998

$\log_{(\sqrt{1}0+2)/3} n \rightsquigarrow \log_{(\sqrt{10}+2)/3} n$

▶**Page 715** last line of answer 11 ——— 28 Nov 1999

6.2.4–8. $\rightsquigarrow$ 6.2.4–8.]

Page 718 line 4 ——— 23 Mar 2006

33–43.] $\rightsquigarrow$ 33–43; N. Blum and K. Mehlhorn, *Theoretical Comp. Sci.* **11** (1980), 303–320.]

▶**Page 722** line 9 ——— 28 Nov 1999

(1991) $\rightsquigarrow$ (1992)

Page 722 bottom line ——— 02 Aug 1998

rA and rX. $\rightsquigarrow$ rA and rX. ▮

Page 726 line 8 of answer 27 ——— 06 Oct 2000

not hard to prove. $\rightsquigarrow$ not hard to prove. Moreover, α turns out to be identical to the constant defined quite differently in 5.2.3–(19); see Karl Dilcher, *Discrete Math.* **145** (1995), 83–93.

▸**Page 726** line 2 of answer 31 ——— 29 Jul 1998

W. Prodinger ⤳ H. Prodinger

Page 726 line 2 of answer 31 ——— 26 Oct 2008

Patricia ⤳ Patrician

▸**Page 727** lines 7 and 8 of answer 34 ——— 28 Nov 1999

$+\cdots$ ⤳ $+\cdots)$ (twice)

Page 728 new answer ——— 08 Mar 1999

45. The probability of {THAT, THE, THIS} before {BUILT, HOUSE, IS, JACK}, {HOUSE, IS, JACK} before {BUILT}, {HOUSE, IS} before {JACK}, {IS} before {HOUSE}, {THIS} before {THAT, THE}, and {THE} before {THAT} is $\frac{3}{7}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{56}$.

Page 732 line 4 of answer 29 ——— 25 Feb 1999

1274.] ⤳ 1274; see also R. Pyke, *Annals of Math. Stat.* **30** (1959), 568–576, Lemma 1.]

▸**Page 733** line 16 ——— 05 Dec 2000

Conversely, The ⤳ Conversely, the

Page 734 new copy replacing lines 1–5 of answer 32 ——— 05 Jan 2002

32. Let

$$s_j = \sum_{k=0}^{j}(b_{k \bmod M}-1).$$

Then, as observed by Svante Janson, we have $c_j=\max_{k\ge j}(s_k-s_j)$, a quantity that is well defined because $\lim_{k\to\infty}s_k=-\infty$.

▸**Page 736** last lines of answer 40 ——— 13 Feb 2005

When $\alpha=1$ ... bounded by 2.12. ⤳ When $\alpha=1$ this is about 2.65, so the standard deviation is bounded by 1.63. [Svante Janson, in *Combinatorics, Prob. and Comp.* **17** (2008), 799–814, has found the asymptotic moments of all orders, also when the search is successful.]

Page 740 line 2 of answer 56 ——— 29 May 1998

to appear ⤳ 37–71

Page 742 lines 7 and 8 of answer 68 ——— 23 Nov 1998

D. E. Knuth, P. Flajolet, ... to appear. ⤳ P. Flajolet, P. V. Poblete, and A. Viola, *Algorithmica* **22** (1998), 490–515; D. E. Knuth, *Algorithmica* **22** (1998), 561–568.

Page 742 line 2 of answer 73 ——— 19 Dec 1999

characters. ⤳ characters. [It was invented as early as 1970 by Alfred L. Zobrist, whose original technical report has been reprinted in *ICCA Journal* **13** (1990), 69–73.]

Page 743 new answer ——— 26 Sep 2007

78. Let $g(x)=\lfloor x/2^k\rfloor \bmod 2^{n-k}$ and $\delta(x,x')=\sum_{b=0}^{2^k-1}[g(x+b)=g(x'+b)]$. Then $\delta(x+1,x'+1)=\delta(x,x')+[g(x+2^k)=g(x'+2^k)]-[g(x)=g(x')]=\delta(x,x')$. Also $\delta(x,0)=(2^k\dot-(x\bmod 2^n))+(2^k\dot-((-x)\bmod 2^n))$ when $0<x<2^n$, where $a\dot-b=\max(a-b,0)$. Therefore $\delta(x,x')=\bigl(2^k\dot-((x-x')\bmod 2^n)\bigr)+\bigl(2^k\dot-((x'-x)\bmod 2^n)\bigr)$ when $x\not\equiv x'$ (modulo 2^n).

Now let $A = \{a \mid 0 < a < 2^n,\ a \text{ odd}\}$ and $B = \{b \mid 0 \le b < 2^k\}$. We want to show that $\sum_{a\in A}\sum_{b\in B}[g(ax+b)=g(ax'+b)] \le R/M = 2^{n-1+k}/2^{n-k} = 2^{2k-1}$ when $0 \le x < x' < 2^n$. And indeed, if $x'-x = 2^p q$ with q odd, then we have

$$\sum_{a\in A}\sum_{b\in B}[g(ax+b)=g(ax'+b)] = \sum_{a\in A}\delta(ax,ax') = 2\sum_{a\in A}\bigl(2^k \dot- ((2^p aq) \bmod 2^n)\bigr)$$

$$= 2^{p+1}\sum_{j=0}^{2^{n-p-1}-1}\bigl(2^k \dot- 2^p(2j+1)\bigr) = 2^{p+1}\sum_{j=0}^{2^{k-p-1}-1}\bigl(2^k - 2^p(2j+1)\bigr)[p<k] = 2^{2k-1}[p<k].$$

[See *Lecture Notes in Computer Science* **1672** (1999), 262–272.]

Page 744 in the third paragraph of answer 1 ———— 04 May 2003

Sperner's Lemma ⇝ *Sperner's Theorem*
Sperner's Lemma ⇝ Sperner's Theorem

▸**Page 744** just before the final display in answer 4 ———— 12 Dec 1999

And If ⇝ And if

Page 745 new material at end of answer 10 ———— 04 Dec 2005

Kirkman wrote his paper in response to a substantially more general problem posed by W. S. B. Woolhouse, namely to find the maximum number of t-element subsets of $\{1,\ldots,n\}$ in which no q-element subset appears more than once; that problem remains unsolved. [See *Lady's and Gentleman's Diary* (1844), 84; (1845), 63–64; (1846), 76, 78; (1847), 62–67.]

▸**Page 747** line 6 of answer 18(e) ———— 25 Mar 2001

$32 \times 8 = 512$ ⇝ $32 \times 16 = 512$

▸**Page 747** last line of answer 18 ———— 04 Jun 2001

also yields on ⇝ also yields an

Page 749 Table 2 . . . . 06 May 1999

In the next edition I plan to give these constants to 36 hexadecimal places, instead of 45 octal places.

Page 750 line 5 ———— 16 Sep 2000

6.3–26, and 6.3–27. ⇝ and 6.3–26.

▸**Page 753** entry for Kronecker delta ———— 14 Aug 1998

1.2.6 ⇝ 1.2.3

▸**Page 755** entry for $\exp x$ ———— 09 Aug 1999

1.2.2 ⇝ 1.2.9

▸**Page 756** entry for $\Im z$ ———— 27 Jan 2001

imaginary part z ⇝ imaginary part of z

Page 757 a new appendix ______ 06 Jan 2011

Beginning with the 27th printing, Volume 3 now has an "Appendix C" analogous to Appendix C of Volume 4A, which lists the page numbers on which named algorithms, theorems, programs, definitions, . . . , can be found. A copy of this two-page appendix can be found online in the author's website. The main index therefore begins now on page 759.

Page 758 quotation for bottom of the page ______ 07 Jan 2011

One of my mathematician friends told me he would be willing
to recognize computer science as a worthwhile field of study
as soon as it contains 1,000 deep theorems.
This criterion should obviously be changed to include algorithms
as well as theorems — say 500 deep theorems and 500 deep algorithms.
But even so, it is clear that computer science today doesn't measure up
to such a test, if "deep" means that a brilliant person would need
many months to discover the theorem or the algorithm.
. . . The potential for "1,000 deep results" is there,
but only perhaps 50 have been discovered so far.

— DONALD E. KNUTH, *Computer Science and Mathematics* (1973)

Page 757 and following ——— 16 Feb 1998

Miscellaneous changes to the existing index of Volume 3 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.